Integration Summary

Introduction to Integration

$$\int 12x^{3} + 4x - 2dx$$

Steps:

1. Add one to the power
2. Divide coefficient by the new power
3. Add c to the end

Introduction to Integration

Two rules you must know

1. Change roots to powers:

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$\sqrt{x^3} = x^{\frac{3}{2}}$$

$$\sqrt[4]{x^5} = x^{\frac{5}{4}}$$

2. Change denominators to numerators:

$$\frac{2x+1}{x^5} = (2x+1)\,x^{-5}$$

$$\frac{5}{17\,(x^2-5)^3} = \frac{5\,(x^2-5)^{-3}}{17}$$

Integration Summary

Introduction to Integration

Kinematics

Distance

$s = \frac{2}{3}x^3 + \frac{3}{2}x^2 + 4x + c$

Differentiation ↓ $+c$ ↑ Integration

Velocity

$v = 2x^2 + 3x + 4$

Differentiation ↓ $+c$ ↑ Integration

Acceleration

$a = 4x + 3$

Integration Skills

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\ln x$	$\frac{1}{x}$
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$-\cos x$	~~−~~$\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$-\operatorname{cosec} x$	~~−~~$\operatorname{cosec} x \cot x$
$-\cot x$	~~−~~$\operatorname{cosec}^2 x$

What you need to know

Integration = anti-differentiation

So, do the **opposite**

Integration Summary

Integration Skills

$f(x)$	$\int f(x)\,dx$
x^n	$\frac{x^{n+1}}{n+1} + c$ $(n \neq -1)$
$\frac{1}{x}$	$\ln\lvert x\rvert + c$
$\frac{f'(x)}{f(x)}$	$\ln\lvert f(x)\rvert + c$

What you need to know

- How to integrate (x^n)
- Fractions & Powers rule doesn't work without a power! ($\frac{1}{x}$)
- When denominator isn't a single x term: Integrate, then divide by the **derivative** of denominator ($\frac{f'(x)}{f(x)}$)

Integration: Composite Functions

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\ln x$	$\frac{1}{x}$
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

1. Integrate Outer
2. Differentiate Inner
3. Divide Outer (1) by Inner (2)

$$\int \sin(e^{2x})\,dx$$

Outer: $\sin(\sim)$ Inner: e^{2x}

1. Integrate 2. Differentiate

$\int$ Outer: $-\cos(\sim)$ $\frac{d}{dx}$ Inner: $2e^{2x}$

Divide: $\frac{-\cos(\sim)}{2e^{2x}} = \frac{-\cos(e^{2x})}{2e^{2x}} + c$

Integration Summary

Integration: Definite Integrals

Find the area enclosed by the graph $y = 4e^{2x}$, the x-axis, and the lines $x = 1$ and $x = 2$

$$\int_1^2 4e^{2x}\,dx$$

Steps

1. Integrate
2. 'High' integral *minus* 'low integral'
3. Simplify and solve

$$\left[\frac{4e^{2x}}{2}\right]_1^2 = [2e^{2x}]_1^2 = \left[2e^{2(2)}\right] - \left[2e^{2(1)}\right]$$

$$= 94.42$$

Integration: Area Under a Graph

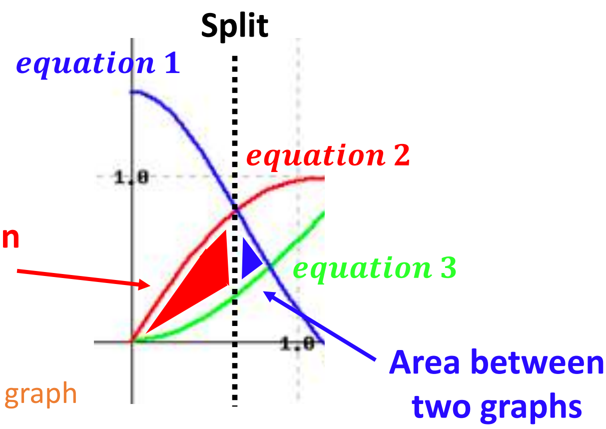
What is the area between the three curves

What you need to know

Steps

1. Integrate
2. 'High' integral *minus* 'low integral'
3. Simplify and solve

Area between two graphs



Rules

1. Crossing x-axis = separate integral
2. Between 2 graphs: high graph *minus* low graph
3. When 3 graphs, split to make 2 graphs

Integration Summary

Integration: Trapezium and Simpson's Rules

What you need to know

Trapezium rule

$$\int_a^b f(x)\,dx \approx \frac{1}{2}h\,[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})]$$

$$where\ h = \frac{b-a}{n}, y_r = f(x_r)$$

Simpson's rule

$$\int_a^b f(x)\,dx \approx \frac{1}{3}h\,[y_0 + y_n + 4(y_1 + y_3 + \cdots + y_{n-1}) + 2(y_2 + y_4 + \cdots + y_{n-2})]$$

$$where\ h = \frac{b-a}{n}, y_r = f(x_r)\ and\ n\ is\ even$$

Integration: Differential Equations

What you need to know

Steps

1. Multiply both sides by dx
2. Put x's on the same side as dx
3. Put y's on the same side as dy
4. Integrate
5. Rearrange to get $y = \sim$

$$\frac{dy}{dx} = \frac{2x^2+1}{8y} \qquad \int 1\,dy = \int \frac{2x^2+1}{8y}\,dx$$

Steps

1. Write a differential with time.
2. Make it = $k\times$[proportional].
3. Rearrange and integrate.

$$\frac{dA}{dt} = kA \qquad \ln|A| = kt + c$$

$$A = A_0 e^{kt}$$

Integration Summary

Integration: Rates of Change

The rate at which the temperature, T is changing in t minutes after sausage rolls are out of the oven is given by the equation:

They are 80^o Celsius coming out of the oven. 5 minutes later they are 60^o Celsius. How hot are they 20 minutes after?

$$\frac{dT}{dt} = -kTt$$

Steps

1. Integrate: $f(t) = \sim$
2. Substitute to find $+c$
3. Substitute to find k
4. Calculate the answer

$$\frac{1}{T}\mathrm{d}T = -kt\ dt \qquad \ln T = \frac{-0.023t^2}{2} + 4.38$$

$$\ln T = \frac{-0.023(20)^2}{2} + 4.38 \qquad = -0.22$$

$= 0.8$ degrees