# VIDEO SUMMARIES: DIFFERENTIATION

## DIFFERENTIATION OVERVIEW

### What you need to know:

Differentiate an equation to get a gradient

We do this by:

1. Multiplying the power by the coefficient
2. Subtracting 1 from the power

- **The gradient at a point**
  - *Differentiate, then substitute x value*
- **Stationary points**
  - *Differentiate, set f'(x) = 0, then solve*
- **Maxima or Minima**
  - *2nd differential, Maxima (-), Minima (+)*
- **Gradient of a tangent line**
  - *Same as gradient at a point*
- **Gradient of a normal line**
  - *Negative reciprocal of tangent (-1/m)*



## ROOTS AND EXPONENTS

### What you need to know

$$x^{\frac{n}{d}} = \sqrt[d]{x^n}$$

**Steps (fraction → root)**

1. Draw root sign
2. Move bottom (denominator) in front of root
3. Leave top (numerator) as power of x

**Steps (root → fraction)**

1. Keep power as top of fraction
2. Put root number on bottom of fraction
3. Get rid of root symbol

**Note:** if no root number denominator =2

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## FRACTIONS AND POWERS

A denominator can become a numerator, if you make the powers negative

$$\frac{5}{17\cdot(x^2-5)^3}=\frac{5\cdot(x^2-5)^{-3}}{17}$$

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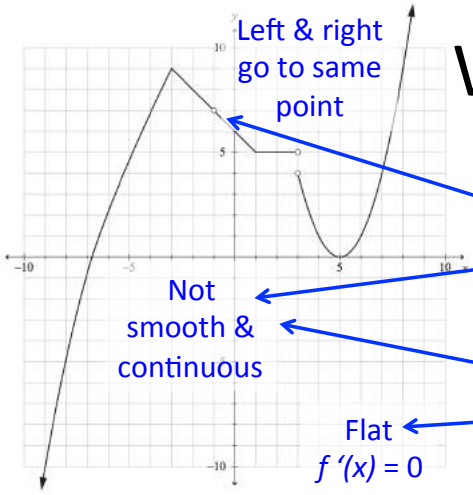
## GRAPHS




### What you need to know

**Part 1:** Equation, *f(x)*
a) read graph values
b) lim *f(x)* as x → a
c) not differentiable

**Part 2:** Differential *f '(x)*
d) not defined
e) stationary points

**Part 3:** Concavity, *f ''(x)*:
f) concave up >0
g) concave down <0
h) point of inflection =0

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## DIFFERENTIATION SKILLS

| $y = f(x)$ | $\frac{dy}{dx} = f'(x)$ |
|---|---|
| $\ln x$ | $\frac{1}{x}$ |
| $e^{ax}$ | $ae^{ax}$ |
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $\sec^2 x$ |
| $\sec x$ | $\sec x \tan x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ |

Natural long (ln) Euler's number (e) Trig functions (sin…)

**What you need to know:**

Use differentiation table to differentiate functions

$f(x) = \ln x$ $f(x) = e^{2x}$ $f(x) = \sin x + \cos x$

$f'(x) = \frac{1}{x}$ $f'(x) = 2e^{2x}$ $f'(x) = \cos x - \sin x$



## CHAIN RULE

For when one differentiable term is inside another differentiable term

For example:

$3(x^2 - 2x)^3$

**Composite Function or Chain Rule**

$(f(g))' = f'(g).g'$

Inner $x^2 - 2x$

Inner ' $2x - 2$

Outer $3(inner)^3$

Outer ' $9(inner)^2$

**Steps**

1. Identify inner and outer terms
2. Differentiate inner and outer
3. Multiply inner ' and outer '
4. Substitute inner

Inner ' x Outer '

$(2x - 2) \cdot 9(inner)^2$

Substitute inner

$(2x - 2) \cdot 9(x^2 - 2x)^2$

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## PRODUCT RULE

For when you have 2 differentiable terms multiplied by each other

For example:

$3x^2(x^3+2)$

f: $3x^2$ g: $x^3+2$

f′: $6x$ g′: $3x^2$

Product Rule:

$$(f.g)' = f.g' + g.f'$$

**Steps**

1. Differentiate both terms
2. Put into equation

$$f.g' + g.f'$$

$$3x^2.3x^2 + 6x(x^3+2)$$

$$9x^4 + 6x^4 + 12x$$



## QUOTIENT RULE

For when you have 2 differentiable terms in a fraction

For example:

$\frac{3x^3+2}{x^5}$

f: $3x^3+2$ g: $x^5$

f′: $9x^2$ g′: $5x^4$

Quotient Rule:

$$\left(\frac{f}{g}\right)' = \frac{g.f' - f.g'}{g^2}$$

**Steps**

1. Differentiate both terms
2. Put into equation

$$\left(\frac{f}{g}\right)' = \frac{x^5.9x^2 - (3x^3+2)5x^4}{\left(x^5\right)^2} = \frac{9x^7 - 15x^7 - 10x^4}{x^{10}}$$

$$= \frac{-6x^7 - 10x^4}{x^{10}} = \frac{-6x^3 - 10}{x^6}$$

# VIDEO SUMMARIES: DIFFERENTIATION

## PARAMETRIC EQUATIONS

For when $x$ and $y$ are explained by a 3rd variable

For example:

$y = 3t^2 \quad x = 6t \quad \frac{dy}{dx} = ?$

Parametric Function

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

**Steps**

1. Differentiate both functions with respect to the 3rd variable
2. Flip one of the answers
3. Multiply together

$$\frac{dy}{dt} = 6t \qquad \frac{dx}{dt} = 6 \qquad \frac{dt}{dx} = \frac{1}{6}$$

$$6t \times \frac{1}{6} = \frac{6t}{6} = t$$



## EXCELLENCE QUESTIONS – PARAMETRIC

The radius increases according to the formula: $r = 1.5t^2$
At what rate is the area inside the ripple increasing after 2 sec?

**Steps**

1. Identify the rate you want to find
2. Identify the rates you have been given
3. Use the parametric equation to find what is missing
4. Find missing differential or repeat as above.
5. Substitute and solve

$$\frac{dA}{dt} \qquad \frac{dr}{dt} = 3t \qquad \frac{dA}{dr} = 2\pi r \qquad A = \pi r^2$$

$$\frac{dA}{dt} = \frac{dr}{dt} \times \frac{dA}{dr}$$

$$\frac{dA}{dt} = 3t \times \pi r^2 = 3t \times \pi (1.5t^2)^2$$

$$\text{At 2 sec } \frac{dA}{dt} = 3(2)\pi(1.5(2)^2)^2 = 678.6 \text{ cm}^2/\text{s}$$

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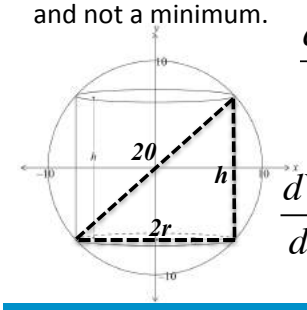
## EXCELLENCE QUESTIONS – MAXIMUMS

A cylinder of height ***h*** cm and radius ***r*** cm is inscribed inside a sphere of radius 10 cm.
Find the value of ***r*** that maximises the volume of the cylinder.
You do not need to prove that the volume is a maximum and not a minimum.

**Steps**

1. Identify what you need
2. Write equation that includes all variables
3. Get in terms of one variable
4. Differentiate and set equal to 0
5. Solve for variable
6. If it is not the answer use solution to solve for other answer




$$\frac{dV}{dr} \quad V = \pi r^2 h \qquad h = \sqrt{400 - 4r^2}$$

$$V = \pi r^2 \sqrt{400 - 4r^2}$$

$$\frac{dV}{dr} = 2\pi r.\sqrt{400 - 4r^2} - \frac{4\pi r^3}{\sqrt{400 - 4r^2}} = 0 \qquad r = \sqrt{\frac{200}{3}} = 8.16 \text{ cm}$$