Complex Numbers Summary

Complex Numbers: Introduction

What you need to know

Gerolamo Cardano

***Changing* and *Solving*:**

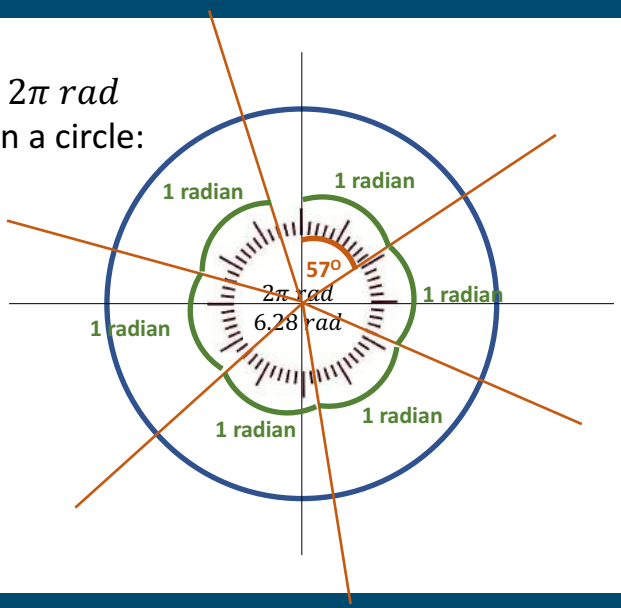
$$x = 0.5 \pm 1.33i$$

2 parts: Real + Imaginary

= complex

Radians

$2\pi\ rad$ in a circle:



1 radian = 57.3 degrees

From degrees → radians:

$\times \pi \div 180$

From radians → degrees:

$\times 180 \div \pi$

What you need to know:

Complex Numbers Summary

Expanding Three Brackets

What you need to know:

1. Expand two sets of brackets with FOIL method
2. Break up third set
3. Expand the brackets
4. Simplify

$$(i+2)(i-5)(i-3)$$

$$(x^2+2x-5x-10)(x-3)$$

$$(x^2-3x-10)(x-3)$$

$$x(x^2-3x-10)-3(x^2-3x-10)$$

$$x^3-3x^2-10x-3x^2+6x+30$$

$$x^3-6x^2-4x+30$$

Factorising

Steps:

1. Write out $a\times c$
2. Write out the difference, b
3. Write out possible factors of $a\times c$
4. Pick the factors with the difference, b
5. Write equation with two factors for b
6. Take out common factor (start and finish)
7. Write as two sets of brackets

Example: $4x^2-5x-6$

1. $a\times c = 4\times -6 = -24$
2. $b = -5$
3. $1\times 24,\ 2\times 12,\ 3\times 8,\ 4\times 6$
4. $3 \ \ and \ \ -8$
5. $4x^2-8x+3x-6$
6. $4x(x-2)+3(x-2)$
7. $(4x+3)(x-2)$

Complex Numbers Summary

Surds

What you need to know

1. What is a surd? A square root that does not simplify E.g. $\sqrt{3}$
2. Answering in Surd form $5 + \sqrt{2.645}^2 i$ $= 5 + \sqrt{7}\, i$
3. Multiplying Surds $(2 + \sqrt{3})(4 + 2\sqrt{3})(\sqrt{3} + 1)$ treat $\sqrt{3}$ as x.
4. Solving equations with Surds $q = \sqrt{12 - 4q} + 4$

$q - 4 = \sqrt{12 - 4q}$ $(q - 4)^2 = \sqrt{12 - 4q}^2$

Factor and Remainder Theorem

What you need to know

In both cases: An expression: $2x^3 - 3x^2 + 4x - 1$, and a 'factor': $x - 2$.

Factor: find another letter | Remainder: find a number

1. Make the number in the factor negative $(x - n)$.
2. **Factor**: Make equation $= 0$. **Remainder**: Make equation $= remainder$
3. Replace x with negative factor
4. Solve.

Complex Numbers Summary

Adding, Subtracting, and Multiplying Complex Numbers

What you need to know

Addition and subtraction

Put **real** and **imaginary** parts together

Multiplication

Treat i (imaginary) terms the same as x.

Replace i^2 with -1 (because $i^2 = -1$)

Complex Fractions

What you need to know

Complex conjugates

= imaginary part becomes opposite ($2 + 3i \rightarrow 2 - 3i$)

or surd

($5 + \sqrt{11} \rightarrow 5 - \sqrt{11}$)

Complex fractions

or surd

When a complex number is on the bottom of a fraction (denominator), multiply top and bottom of fraction with complex conjugate.

(or surd conjugate).

$$\frac{2}{2+3i} \cdot \frac{2-3i}{2-3i}$$

Complex Numbers Summary

Finding Solutions

What you need to know

Finding polynomial solutions

$2x^3 - 16x^2 + Bx - 52 = 0.$

Create three sets of brackets, find what is in front of the remaining x.

Complex conjugates

E.g. $(2x+c)(x-3+2i)(x-3-2i)$

If a complex number is one solution, it's conjugate is also a solution.

E.g. If $x = 5 + 3i$ is onr solution, then $x = 5 - 3i$ is also a solution.

Complex polynomial solutions

Expand. Simplify. Compare to the original equation to solve.

Argand Diagrams

What you need to know

Drawing Argand diagrams

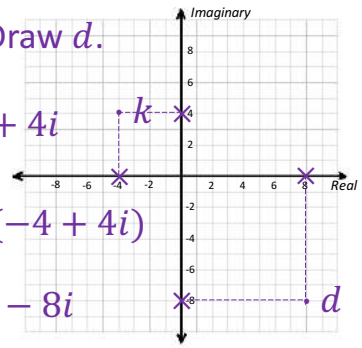
Plot real number, then imaginary number, then join the points.

Reading Argand diagrams

Read real number, then imaginary number, then join the parts.

$d = -2k$. Draw d.

$k = -4 + 4i$



Equations with Argand diagrams

Read complex numbers, do calculation, draw complex number.

$-2k = -2(-4 + 4i)$

$= 8 - 8i$

Complex Numbers Summary

Rectangular and Polar Form

What you need to know

Changing from rectangular to polar form ($a + bi \rightarrow r\ cis\ \theta$) E.g. $q = 4 + 3i$

Find modulus, r, using Pythagoras: $r = \sqrt{a^2 + b^2}$ $r = \sqrt{4^2 + 3^2} = 5$

Find argument, θ, using trigonometry: $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ $\theta = \tan^{-1}\left(\frac{3}{4}\right) = 0.64$

Changing from polar to rectangular form ($r\ cis\ \theta \rightarrow a + bi$) $q = 5\ cis\ 0.64$

Write out: $z = r\cos\theta + r\sin\theta\ \ i$ $q = 5\ cos\ 0.64 + 5\ sin\ 0.64\ i$

Find a using: $r\cos\theta$. Find b using: $r\sin\theta$ $q = 4 + 3i$

Argument and Locus

What you need to know

Finding an argument, $arg(z)$:

Use the formula: $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

When:

- $arg(z) = \frac{\pi}{4}, Re(z) = Im(z)$
- $arg(z) = \frac{\pi}{2}, Re(z) = 0$
- $arg(z) = 0$ or $\pi, Im(z) = 0$

Finding the equation of a locus $|z|$:

1. Replace z with $x + yi$
2. Combine real & imaginary terms
3. $|modulus| = r = \sqrt{a^2 + b^2}$
4. Expand and simplify

Equations to look out for:
- $x^2 + y^2 = {\sim}^2$
- $y = {\sim}(x)$

Complex Numbers Summary

Powers, Roots, Multiplication and Division

What you need to know

Steps $(\times)$:
- Multiply r
- Write cis
- Add θ

Steps $(\div)$:
- Divide r
- Write cis
- Subtract θ

Steps (z^x):
- r^x
- Write cis
- $\theta \times$ power

Steps $(\sqrt[x]{z})$:
- $\sqrt[x]{r}$
- Write cis
- $\theta / root$

Multiple solutions: add $\frac{2\pi}{root}$ to the angle

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