

VIDEO SUMMARIES: CALCULUS

DIFFERENTIATION

What you need to know:

1. Multiply coefficient by power
2. Subtract 1 from power

Notation:

- y is the same as $f(x)$
- You differentiate to go from a function to a gradient
- When differentiating y becomes $\frac{dy}{dx}$
- $f(x)$ becomes $f'(x)$

FREE tutorial videos at www.learncoach.co.nz



FINDING A GRADIENT AT A POINT

What you need to know:

1. Differentiate
2. Substitute in x value
3. Solve

Find the gradient of the curve $y = 3x^2 + 2x - 5$ at the point $x = -2$.

$$\frac{dy}{dx} = 6x + 2$$

$$\frac{dy}{dx} = 6(-2) + 2$$

$$\frac{dy}{dx} = -10$$

FREE tutorial videos at www.learncoach.co.nz



VIDEO SUMMARIES: CALCULUS

FINDING A POINT FROM A GRADIENT

What you need to know:

1. Differentiate
2. Substitute gradient in
3. Solve for x
4. Find y
5. Give Coordinates

Given curve: $y = 4x^2 + 13x - 4$
Find the point where the curve has a gradient of -3.

$$\frac{dy}{dx} = 8x + 13$$

$$-3 = 8x + 13$$

$$-16 = 8x$$

$$-2 = x$$

$$y = 4(-2)^2 - 6(-2) + 7$$

$$(-2, 35)$$

FREE tutorial videos at www.learncoach.co.nz



FINDING THE EQUATION OF A TANGENT

What you need to know:

1. Differentiate
2. Find gradient
3. Find c
4. Put m and c only into equation

Find tangent of $y = 2x^2 + 3x + 2$
through point $(-3, 11)$.

$$\frac{dy}{dx} = 4x + 3$$

$$\frac{dy}{dx} = 4(-3) + 3$$

$$11 = 4(-3) + c$$

$$-16 = c$$

$$y = -9x - 16$$

FREE tutorial videos at www.learncoach.co.nz



VIDEO SUMMARIES: CALCULUS

RATE OF CHANGE

What you need to know:

1. Differentiate
2. Substitute time in
3. Solve

The volume, V , of a cube changes as the length x of the sides changes. If the sides are changing at a rate of $1\text{cm}/\text{sec}^2$, the equation for the changing volume of the cube can be given by: $V = t^3$

Find the rate of change of the volume, after five seconds.

$$\frac{dV}{dt} = 3t^2$$

$$\frac{dV}{dt} = 3(5)^2$$

$$\frac{dV}{dt} = 75 \text{ cm}^3 / \text{sec}$$

FREE tutorial videos at www.learncoach.co.nz



TURNING POINTS

What you need to know:

1. Differentiate
2. Set gradient to 0
3. Solve for x
4. Find y
5. Give coordinates
6. Differentiate
7. Differentiate Again
8. Positive = minimum
Negative = maximum

Find the coordinates of the turning points of the curve $y = -2x^2 - 4x + 1$ and determine their nature.

$$\frac{dy}{dx} = -4x - 4$$

$$0 = -4x - 4$$

$$x = -1$$

$$y = -2(-1)^2 - 4(-1) + 1$$

$$y = 3$$

$$(-1, 3)$$

$$\frac{d^2y}{dx^2} = -4$$

$(-1, 3)$ is a maximum

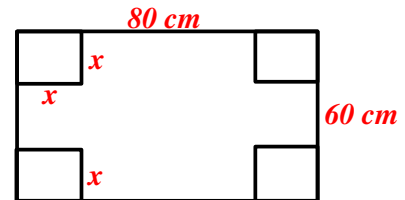
FREE tutorial videos at www.learncoach.co.nz



VIDEO SUMMARIES: CALCULUS

TURNING POINTS

An open box is to be made from a piece of cardboard measuring 80 cm x 60 cm.
Calculate the maximum volume of the box.



What you need to know:

1. Write two equations $V = h + w + d$ $h = x$
2. Combine equations $V = x(80 - 2x)(60 - 2x)$
3. Simplify $V = 4800x - 280x^2 + 4x^3$
4. Differentiate $\frac{dV}{dx} = 4800 - 560x + 12x^2$
5. Set equal to zero $0 = 4800 - 560x + 12x^2$
6. Solve $x = 35.4$ or 11.3 *Doesn't work as width would be negative*
7. Find other unknown $V = 35.4(80 - 70.8)(60 - 70.8)$
 $V = 11.3(80 - 22.6)(60 - 22.6)$

FREE tutorial videos at www.learncoach.co.nz



INTEGRATION

What you need to know: $\int 6x^2 - 3x + 4 \quad dx$

1. Add one to the power
2. Divide coefficient by the new power
3. Add c to the end

$$\frac{6}{3}x^3$$

FREE tutorial videos at www.learncoach.co.nz



VIDEO SUMMARIES: CALCULUS

FINDING AN EQUATION FROM A GRADIENT

What you need to know: $f'(x) = 12x^3 + 3x^2 - 5$
through the point (2,40)

1. Integrate

$$f(x) = 3x^4 + x^3 - 5x + c$$

2. Substitute x and y in

$$40 = 3(2)^4 + (2)^3 - 5(2) + c$$

3. Solve for c

$$40 = 46 + c$$

4. Put c back into equation

$$-6 = c$$

$$f(x) = 3x^4 + x^3 - 5x - 6$$

FREE tutorial videos at www.learncoach.co.nz



KINEMATICS

I Am Very Sexy
→ Int → Int →

A train starts from rest. Its acceleration t seconds after it starts is given

by $a = \frac{1}{4}(20 - t) \text{ m s}^{-2}$ What distance does the train cover in the first 30 seconds?

Steps:

1. Write IAVS

$$a = 5 - \frac{t}{4}$$

2. Integrate

$$v = 5t - \frac{t^2}{8}$$

3. Find c

$$0 = 5(0) - \frac{(0)^2}{8} + c$$

4. Integrate

$$c = 0$$

5. Find c

$$v = 5t - \frac{t^2}{8}$$

6. Substitute and solve

$$s = \frac{5t^2}{2} - \frac{t^3}{24} + c$$

$$0 = \frac{5(0)^2}{2} - \frac{(0)^3}{24} + c$$
$$c = 0$$

$$s = \frac{5(30)^2}{2} - \frac{(30)^3}{24}$$

FREE tutorial videos at www.learncoach.co.nz

