ALGEBRA

APPLY ALGEBRAIC PROCEDURES IN SOLVING EQUATIONS
4 CREDITS (91027)

THE SKILLS YOU NEED TO KNOW:

SOLVING LINEAR EQUATIONS p 17

1. Multiply away any fractions
2. Expand any brackets
3. Put x's on the same side
4. Put numbers on the opposite side
5. Simplify
6. Divide by the last number

SOLVING EXPONENTS p 23

1. Simplify with the following rules:
   - If the base is the same add the powers. e.g. $x^m x^n = x^{m+n}$
   - If the exponent is raised to a power multiply the exponent and power. e.g. $(x^m)^n = x^{m \times n}$
2. Trial with 2
3. Trial with 3 Etc.
4. If the power is even numbered you have both +ve and -ve answers

SUBSTITUTION p 28

1. Put the numbers in place of the letters
2. Simplify or solve the term

REARRANGING p 28

1. Rearrange to make sure that the subject is a numerator not a denominator
2. Get rid of all other symbols/ numbers around the subject

SOLVING QUADRATICS p 19

1. Make the equation = 0
2. Factorise
3. Make each bracket = 0
4. Solve for x (twice)

SOLVING FRACTIONS WITH QUADRATICS p 21

1. Factorise top and bottom
2. Cancel common factors
3. Multiply both sides by the bottom
4. Solve as a linear equation

SOLVING INEQUALITIES p 25

1. Multiply away any fractions
2. Expand any brackets
3. Put x's on the same side
4. Put numbers on the opposite side
5. Simplify
6. Divide by the last number (*if negative reverse the sign*)

SIMULTANEOUS EQUATIONS p 32

1. Write out the two equations if they haven't been given
2. Rearrange one equation so that a variable is the subject
3. Substitute into the other equation and solve for one variable
4. Substitute answer into rearranged equation and solve for other variable

WORD QUESTIONS p 35

1. Give each variable a letter, the letter will ALWAYS represent a number.
2. If calculations are needed, put the values from the question into the equations
3. There is now enough info. to either solve the problem or explain your answer

Note: Problems may rely on knowledge from earlier algebra sections

SIMPLIFYING ALGEBRAIC EQUATIONS

STEPS:	EXAMPLE: $9x^2y - 4xy^2 + 2x^2y + xy^2 - x$
1. Identify terms which are common **2.** Put common terms together	$\underline{9x^2y} - \mathbf{4xy^2} + \underline{2x^2y} + \mathbf{xy^2} - x$ $\underline{11x^2y} - \mathbf{3xy^2} - x$

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OLD NCEA QUESTIONS

Simplify the following equations:

1. $4ab^2 + 3a^2b - a^2b$

2. $3xy^2 + 2x^2y - xy^2$

3. $3ab^2 + 2a^2b - ab^2$

PRACTICE QUESTIONS

Simplify the following equations:

4. $5x + 3 - 6 + 8x$

5. $6a - 3b - 9a + 11b$

6. $3w - 8e - 19w + 15e$

7. $7a - 9a + 11$

8. $3x - 12 - 11x - 66$

9. $4x^2 + 3y + 5 + 6x^2 - y$

10. $2a + 4b - 3c - 6b - a$

11. $4xy^2 - 2y + 4 - 7xy^2 + 6$

12. $-3a^3 + 6a^2 + 12 - 3a^2 + 5a^3$

13. $5b + 6c - 4a + 3b + 6c - a$

ANSWERS

NCEA

1. $4ab^2 + \boldsymbol{3a^2b} - \boldsymbol{a^2b}$
$4ab^2 + 2a^2b$ (Achieved)

2. $3\boldsymbol{xy^2} + 2x^2y - \boldsymbol{xy^2}$
$2xy^2 + 2x^2y$ (Achieved)

3. $\boldsymbol{3ab^2} + 2a^2b - \boldsymbol{ab^2}$
$2ab^2 + 2a^2b$ (Achieved)

PRACTICE

4. $\boldsymbol{5x} + 3 - 6 + \boldsymbol{8x}$
$13x - 3$ (Achieved)

5. $6a - \boldsymbol{3b} - 9a + \boldsymbol{11b}$
$-3a + 8b$ (Achieved)

6. $3w - \boldsymbol{8e} - 19w + \boldsymbol{15e}$
$-16w + 7e$ (Achieved)

7. $\boldsymbol{7a} - \boldsymbol{9a} + 11$
$-2a + 11$ (Achieved)

8. $\boldsymbol{3x} - 12 - \boldsymbol{11x} - 66$
$-8x - 78$ (Achieved)

9. $\boldsymbol{4x^2} + \underline{3y} + 5 + \boldsymbol{6x^2} - \underline{y}$
$10x^2 + 2y + 5$ (Achieved)

10. $\boldsymbol{2a} + \underline{4b} - 3c - \underline{6b} - \boldsymbol{a}$
$a - 2b - 3c$ (Achieved)

11. $\boldsymbol{4xy^2} - 2y + \underline{4} - \boldsymbol{7xy^2} + \underline{6}$
$-3xy^2 - 2y + 10$ (Achieved)

12. $-\boldsymbol{3a^3} + \underline{6a^2} + 12 - \underline{3a^2} + \boldsymbol{5a^3}$
$2a^3 + 3a^2 + 12$ (Achieved)

13. $\boldsymbol{5b} + \underline{6c} - 4a + \boldsymbol{3b} + \underline{6c} - a$
$8b + 12c - 5a$ (Achieved)

Study Tip:

Quality over Quantity

Great news!
*It is the **quality** of **study** that makes more of an **impact** than the quantity.*
***Interesting fact:** the amount of time students spend on homework has risen over the last 3 decades – but the level of educational attainment has not risen with the increased workload.*
Work Smarter, not harder.

EXPANDING BRACKETS

STEPS:	EXAMPLE: $5x(2-x)$
1. Multiply the term outside the brackets by everything inside the brackets	$5x \times 2 - 5x \times x$
2. Simplify	$10x - 5x^2$

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OLD NCEA QUESTIONS

Expand the following equations:

1. $2x(3-x)$

2. $2(x-7)$

3. $2(x-2)$

4. $5(x-3)$

5. $2(x-1)-3(x+2)$

6. $3(x+4)-2(x+5)$

PRACTICE QUESTIONS

Expand the following equations:

7. $4(y+3)+2(x-2)$

8. $6(x+11)-4(x+3)$

9. $8(x+9)$

10. $3x(x-15)$

11. $4(t+12)$

12. $6(7-5x)$

13. $7(12-n)$

14. $x(4-x)+3(2x-5)$

15. $2(x-3)+2(4-x)$

16. $3x(9-4x)$

ANSWERS

NCEA

1. $2x(3-x)$
$2x\times 3-2x\times x$
$6x-2x^2$ (Achieved)

2. $2(x-7)$
$2\times x+2\times -7$
$2x-14$ (Achieved)

3. $2(x-2)$
$2\times x+2\times -2$
$2x-4$ (Achieved)

4. $5(x-3)$
$5\times x+5\times -3$
$5x-15$ (Achieved)

5. $2(x-1)-3(x+2)$
$2\times x+2\times -1-3\times x-3\times 2$
$2x-2-3x-6$
$-x-8$ (Achieved)

6. $3(x+4)-2(x+5)$
$3\times x+3\times 4-2\times x-2\times 5$
$3x+12-2x-10$
$x+2$ (Achieved)

PRACTICE

7. $4(y+3)+2(x-2)$
$4\times y+4\times 3+2\times x+2\times -2$
$4y+12+2x-4$
$4y+2x+8$ (Achieved)

8. $6(x+11)-4(x+3)$
$6\times x+6\times 11-4\times x-4\times 3$
$6x+66-4x-12$
$2x+54$ (Achieved)

9. $8(x+9)$
$8\times x+8\times 9$
$8x+72$ (Achieved)

10. $3x(x-15)$
$3x\times x+3x\times -15$
$3x^2-45x$ (Achieved)

11. $4(t+12)$
$4\times t+4\times 12$
$4t+48$ (Achieved)

12. $6(7-5x)$
$6\times 7+6\times -5x$
$42-30x$ (Achieved)

13. $7(12-n)$
$7\times 12+7\times -n$
$84-7n$ (Achieved)

14. $x(4-x)+3(2x-5)$
$x\times 4+x\times -x+3\times 2x+3\times -5$
$4x-x^2+6x-15$
$10x-x^2-15$ (Achieved)

15. $2(x-3)+2(4-x)$
$2\times x+2\times -3+2\times 4+2\times -x$
$2x-6+8-2x$
2 (Achieved)

16. $3x(9-4x)$
$3x\times 9+3x\times -4x$
$27x-12x^2$ (Achieved)

EXPANDING TWO BRACKETS

STEPS: FOIL	EXAMPLE: $(3x-4)(x+2)$
1. Multiply the **F**irst two terms of each bracket	$(\mathbf{3x}-4)(\mathbf{x}+2) \Rightarrow \mathbf{3x^2}$
2. Multiply the **O**utside two terms of each bracket	$(\mathbf{3x}-4)(x+\mathbf{2}) \Rightarrow \mathbf{6x}$
3. Multiply the **I**nside two terms of each bracket	$(3x-\mathbf{4})(\mathbf{x}+2) \Rightarrow \mathbf{-4x}$
4. Multiply the **L**ast two terms of each bracket	$(3x-\mathbf{4})(x+\mathbf{2}) \Rightarrow \mathbf{8}$
5. Write each answer together	$3x^2+6x-4x-8$
6. Simplify (usually by adding the two middle terms)	$3x^2+2x-8$

For a complete tutorial on this topic visit www.learncoach.co.nz

OLD NCEA QUESTIONS

Expand the following equations:

1. $(2x+5)(x-2)$

2. $(2x+3)(x-4)$

3. $(x+5)(x-7)$

4. $(1-2x)(x+3)$

5. $(2x-1)(x+3)$

6. $(4x-5)(x+2)$

PRACTICE QUESTIONS

Expand the following equations:

7. $(3j+7)(9-j)$

8. $(4n+1)^2$

9. $(4x+3)(x-4)$

10. $(x+8)(x-9)$

11. $(x+8)^2$

12. $(6x-7)(x-2)$

13. $(2x+9)(4x-7)$

14. $(x+4)(x-4)$

15. $(3x+2)(x+1)$

16. $(x-7)(3+x)$

ANSWERS

NCEA

1. $(\mathbf{2}x+5)(\boldsymbol{x}-2) \Rightarrow \mathbf{2}\boldsymbol{x}^2$ **F**
$(\mathbf{2}x+5)(x-\mathbf{2}) \Rightarrow \mathbf{-4}\boldsymbol{x}$ **O**
$(2x+\mathbf{5})(\boldsymbol{x}-2) \Rightarrow \mathbf{5}\boldsymbol{x}$ **I**
$(2x+\mathbf{5})(x-\mathbf{2}) \Rightarrow \mathbf{-10}$ **L**
$2x^2-4x+5x-10$
$2x^2+x-10$ (Achieved)

2. $(\mathbf{2}\boldsymbol{x}+3)(\boldsymbol{x}-4) \Rightarrow \mathbf{2}\boldsymbol{x}^2$ **F**
$(\mathbf{2}\boldsymbol{x}+3)(x-\mathbf{4}) \Rightarrow \mathbf{-8}\boldsymbol{x}$ **O**
$(2x+\mathbf{3})(\boldsymbol{x}-4) \Rightarrow \mathbf{3}\boldsymbol{x}$ **I**
$(2x+\mathbf{3})(x-\mathbf{4}) \Rightarrow \mathbf{-12}$ **L**
$2x^2-8x+3x-12$
$2x^2-5x-12$ (Achieved)

3. $(\boldsymbol{x}+5)(\boldsymbol{x}-7) \Rightarrow \boldsymbol{x}^2$ **F**
$(\boldsymbol{x}+5)(x-\mathbf{7}) \Rightarrow \mathbf{-7}\boldsymbol{x}$ **O**
$(x+\mathbf{5})(\boldsymbol{x}-7) \Rightarrow \mathbf{5}\boldsymbol{x}$ **I**
$(x+\mathbf{5})(x-\mathbf{7}) \Rightarrow \mathbf{-35}$ **L**
$x^2-7x+5x-35$
$x^2-2x-35$ (Achieved)

4. $(\mathbf{1}-2x)(\boldsymbol{x}+3) \Rightarrow \boldsymbol{x}$ **F**
$(\mathbf{1}-2x)(x+\mathbf{3}) \Rightarrow \mathbf{3}$ **O**
$(1-\mathbf{2}\boldsymbol{x})(\boldsymbol{x}+3) \Rightarrow \mathbf{-2}\boldsymbol{x}^2$ **I**
$(1-\mathbf{2}\boldsymbol{x})(x+\mathbf{3}) \Rightarrow \mathbf{-6}\boldsymbol{x}$ **L**
$x+3-2x^2-6x$
$3-5x-2x^2$ (Achieved)

5. $(\mathbf{2}\boldsymbol{x}-1)(\boldsymbol{x}+3) \Rightarrow \mathbf{2}\boldsymbol{x}^2$ **F**
$(\mathbf{2}\boldsymbol{x}-1)(x+\mathbf{3}) \Rightarrow \mathbf{6}\boldsymbol{x}$ **O**
$(2x-\mathbf{1})(\boldsymbol{x}+3) \Rightarrow \mathbf{-}\boldsymbol{x}$ **I**
$(2x-\mathbf{1})(x+\mathbf{3}) \Rightarrow \mathbf{-3}$ **L**
$2x^2+6x-x-3$
$2x^2+5x-3$ (Achieved)

6. $(\mathbf{4}\boldsymbol{x}-5)(\boldsymbol{x}+2) \Rightarrow \mathbf{4}\boldsymbol{x}^2$ **F**
$(\mathbf{4}\boldsymbol{x}-5)(x+\mathbf{2}) \Rightarrow \mathbf{8}\boldsymbol{x}$ **O**
$(4x-\mathbf{5})(\boldsymbol{x}+2) \Rightarrow \mathbf{-5}\boldsymbol{x}$ **I**
$(4x-\mathbf{5})(x+\mathbf{2}) \Rightarrow \mathbf{-10}$ **L**
$4x^2+8x-5x-10$
$4x^2+3x-10$ (Achieved)

PRACTICE

7. $(\mathbf{3}\boldsymbol{j}+7)(\mathbf{9}-j) \Rightarrow \mathbf{27}\boldsymbol{j}$
$(3j+7)(9-\boldsymbol{j}) \Rightarrow \mathbf{-3}\boldsymbol{j}^2$
$(3j+\mathbf{7})(\mathbf{9}-j) \Rightarrow \mathbf{63}$
$(3j+\mathbf{7})(9-\boldsymbol{j}) \Rightarrow \mathbf{-7}\boldsymbol{j}$
$27j-3j^2+63-7j$
$20j-3j^2+63$ (Achieved)

8. $(\mathbf{4}\boldsymbol{n}+1)(\mathbf{4}\boldsymbol{n}+1) \Rightarrow \mathbf{16}\boldsymbol{n}^2$
$(\mathbf{4}\boldsymbol{n}+1)(4n+\mathbf{1}) \Rightarrow \mathbf{4}\boldsymbol{n}$
$(4n+\mathbf{1})(\mathbf{4}\boldsymbol{n}+1) \Rightarrow \mathbf{4}\boldsymbol{n}$
$(4n+\mathbf{1})(4n+\mathbf{1}) \Rightarrow \mathbf{1}$
$16n^2+4n+4n+1$
$16n^2+8n+1$ (Achieved)

9. $(\mathbf{4}\boldsymbol{x}+3)(\boldsymbol{x}-4) \Rightarrow \mathbf{4}\boldsymbol{x}^2$
$(\mathbf{4}\boldsymbol{x}+3)(x-\mathbf{4}) \Rightarrow \mathbf{-16}\boldsymbol{x}$
$(4x+\mathbf{3})(\boldsymbol{x}-4) \Rightarrow \mathbf{3}\boldsymbol{x}$
$(4x+\mathbf{3})(x-\mathbf{4}) \Rightarrow \mathbf{-12}$
$4x^2-16x+3x-12$
$4x^2-13x-12$ (Achieved)

10. $(\boldsymbol{x}+8)(\boldsymbol{x}-9) \Rightarrow \boldsymbol{x}^2$
$(\boldsymbol{x}+8)(x-\mathbf{9}) \Rightarrow \mathbf{-9}\boldsymbol{x}$
$(x+\mathbf{8})(\boldsymbol{x}-9) \Rightarrow \mathbf{8}\boldsymbol{x}$
$(x+\mathbf{8})(x-\mathbf{9}) \Rightarrow \mathbf{-72}$
$x^2-9x+8x-72$
x^2-x-72 (Achieved)

11. $(\boldsymbol{x}+8)(\boldsymbol{x}+8) \Rightarrow \boldsymbol{x}^2$
$(\boldsymbol{x}+8)(x+\mathbf{8}) \Rightarrow \mathbf{8}\boldsymbol{x}$
$(x+\mathbf{8})(\boldsymbol{x}+8) \Rightarrow \mathbf{8}\boldsymbol{x}$
$(x+\mathbf{8})(x+\mathbf{8}) \Rightarrow \mathbf{64}$
$x^2+8x+8x+64$
$x^2+16x+64$ (Achieved)

12. $(\mathbf{6}\boldsymbol{x}-7)(\boldsymbol{x}-2) \Rightarrow \mathbf{6}\boldsymbol{x}^2$
$(\mathbf{6}\boldsymbol{x}-7)(x-\mathbf{2}) \Rightarrow \mathbf{-12}\boldsymbol{x}$
$(6x-\mathbf{7})(\boldsymbol{x}-2) \Rightarrow \mathbf{-7}\boldsymbol{x}$
$(6x-\mathbf{7})(x-\mathbf{2}) \Rightarrow \mathbf{14}$
$6x^2-12x-7x+14$
$6x^2-19x+14$ (Achieved)

13. $(\mathbf{2}\boldsymbol{x}+9)(\mathbf{4}\boldsymbol{x}-7) \Rightarrow \mathbf{8}\boldsymbol{x}^2$
$(\mathbf{2}\boldsymbol{x}+9)(4x-\mathbf{7}) \Rightarrow \mathbf{-14}\boldsymbol{x}$
$(2x+\mathbf{9})(\mathbf{4}\boldsymbol{x}-7) \Rightarrow \mathbf{36}\boldsymbol{x}$
$(2x+\mathbf{9})(4x-\mathbf{7}) \Rightarrow \mathbf{-63}$
$8x^2-14x+36x-63$
$8x^2+22x-63$ (Achieved)

14. $(\boldsymbol{x}+4)(\boldsymbol{x}-4) \Rightarrow \boldsymbol{x}^2$
$(\boldsymbol{x}+4)(x-\mathbf{4}) \Rightarrow \mathbf{-4}\boldsymbol{x}$
$(x+\mathbf{4})(\boldsymbol{x}-4) \Rightarrow \mathbf{4}\boldsymbol{x}$
$(x+\mathbf{4})(x-\mathbf{4}) \Rightarrow \mathbf{-16}$
$x^2-4x+4x-16$
x^2-16 (Achieved)

15. $(\mathbf{3}\boldsymbol{x}+2)(\boldsymbol{x}+1) \Rightarrow \mathbf{3}\boldsymbol{x}^2$
$(\mathbf{3}\boldsymbol{x}+2)(x+\mathbf{1}) \Rightarrow \mathbf{3}\boldsymbol{x}$
$(3x+\mathbf{2})(\boldsymbol{x}+1) \Rightarrow \mathbf{2}\boldsymbol{x}$
$(3x+\mathbf{2})(x+\mathbf{1}) \Rightarrow \mathbf{2}$
$3x^2+3x+2x+2$
$3x^2+5x+2$ (Achieved)

16. $(\boldsymbol{x}-7)(\mathbf{3}+x) \Rightarrow \mathbf{3}\boldsymbol{x}$
$(\boldsymbol{x}-7)(3+\boldsymbol{x}) \Rightarrow \boldsymbol{x}^2$
$(x-\mathbf{7})(\mathbf{3}+x) \Rightarrow \mathbf{-21}$
$(x-\mathbf{7})(3+\boldsymbol{x}) \Rightarrow \mathbf{-7}\boldsymbol{x}$
$3x+x^2-21-7x$
$x^2-4x-21$ (Achieved)

SIMPLIFYING ALGEBRAIC FRACTIONS

STEPS:	EXAMPLE: $\frac{8x^2}{2x^5}$
1. Simplify the number fraction	$\frac{4x^2}{x^5}$
2. To simplify the x values, subtract the smaller power from the larger power and remove the x value with the small power.	$\frac{4\cancel{x^2}}{x^{5-2}} = \frac{4}{x^3}$

For a complete tutorial on this topic visit www.learncoach.co.nz

OLD NCEA QUESTIONS

Simplify the following fractions:

1. $\frac{8x^2}{4x}$

2. $\frac{12a^2}{3a}$

3. $\frac{10a^2}{2a}$

4. $\frac{9x^5}{12x^3}$

PRACTICE QUESTIONS

Simplify the following fractions:

5. $\frac{4a^7}{7a^3}$

6. $\frac{11x^9}{x^4}$

7. $\frac{12x^8}{4x^3}$

8. $\frac{3e^6}{6e^5}$

9. $\frac{9x^4}{3x^7}$

10. $\frac{12x^3}{6x^2}$

11. $\frac{24x^7}{4x^4}$

12. $\frac{5x^3}{10x^4}$

13. $\frac{2x^5}{8x^4}$

14. $\frac{12x^9}{4x^9}$

ANSWERS

NCEA

1. $\frac{8x^2}{4x} = \frac{2x^2}{x} = 2x^{2-1} = 2x$ (Achieved)

2. $\frac{12a^2}{3a} = \frac{4a^2}{a} = 4a^{2-1} = 4a$ (Achieved)

3. $\frac{10a^2}{2a} = \frac{5a^2}{a} = 5a^{2-1} = 5a$ (Achieved)

4. $\frac{9x^5}{12x^3} = \frac{3x^5}{4x^3} = \frac{3x^{5-3}}{4} = \frac{3x^2}{4}$ (Achieved)

PRACTICE

5. $\frac{4a^7}{7a^3} = \frac{4a^7}{7a^3} = \frac{4a^{7-3}}{7} = \frac{4}{7}a^4$ (Achieved)

6. $\frac{11x^9}{x^4} = \frac{11x^9}{x^4} = 11x^{9-4} = 11x^5$ (Achieved)

7. $\frac{12x^8}{4x^3} = \frac{3x^8}{x^3} = 3x^{8-3} = 3x^5$ (Achieved)

8. $\frac{3e^6}{6e^5} = \frac{e^6}{2e^5} = \frac{e^{6-5}}{2} = \frac{e}{2}$ (Achieved)

9. $\frac{9x^4}{3x^7} = \frac{3x^4}{x^7} = \frac{3}{x^{7-4}} = \frac{3}{x^3}$ (Achieved)

10. $\frac{12x^3}{6x^2} = \frac{2x^3}{x^2} = 2x^{3-2} = 2x$ (Achieved)

11. $\frac{24x^7}{4x^4} = \frac{6x^7}{x^4} = 6x^{7-4} = 6x^3$ (Achieved)

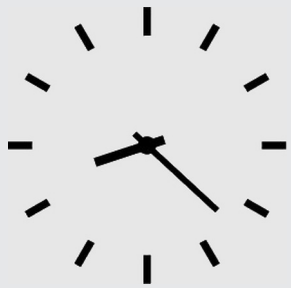
12. $\frac{5x^3}{10x^4} = \frac{x^3}{2x^4} = \frac{1}{2x^{4-3}} = \frac{1}{2x}$ (Achieved)

13. $\frac{2x^5}{8x^4} = \frac{x^5}{4x^4} = \frac{x^{5-4}}{4} = \frac{x}{4}$ (Achieved)

14. $\frac{12x^9}{4x^9} = \frac{3x^9}{x^9} = 3x^{9-9} = 3$ (Achieved)

Study Tip:

The Beginning and the End



***Fact**: most information that sticks in our brains is learnt either at the **beginning** or **end** of a study session.*
('Beginning' and 'End' are about 15 - 20 mins each.)
***Solution**: Cut out the middle part!*
So each hour:
***Study** 35 mins ⟹ **Break** 5 mins ⟹ **Revise** 10 mins ⟹ **Rest** 10 mins*

FACTORISING - COMMON FACTOR

STEPS:	EXAMPLE: $2xy^2 - 8x^2y$
1. Write out a set of brackets	$(\quad)$
2. Identify the greatest common number in each term. Put this outside the brackets	$2(\quad)$
3. Identify any common letters in each term. Put these outside the brackets	$2xy(\quad)$
4. Put the original term into the brackets with the common factors cancelled out (powers cancel too)	$2xy(\cancel{2x}\,y^{\cancel{2}} - \cancel{8}(4)x^{\cancel{2}}\,\cancel{y})$
5. Simplify (anything completely crossed out will have a 1 in its place)	$2xy(y - 4x)$

For a complete tutorial on this topic visit www.learncoach.co.nz

OLD NCEA QUESTIONS

Factorise the following equations:

1. $ab^2 + a^2b$

2. $2ab^2 + 2a^2b$

PRACTICE QUESTIONS

Factorise the following equations:

3. $3n^3 - n$

4. $6x^2 - 18x$

5. $7x - 49x^2$

6. $8d^7 - 2d^3$

7. $6x^5y^3 - 9x^3y^6$

8. $3ab^2 + 6b^2$

9. $24g - 18g^3$

10. $x^3y^2 + 3xy^4$

11. $15a^2b^3 - 10ab^5$

12. $3xy^3 - 4x^3y^2$

ANSWERS

NCEA

1. $ab^2 + a^2b$

$ab(\quad)$

$ab(\cancel{a}b^{\cancel{2}} + a^{\cancel{2}}\cancel{b})$

$ab(b + a)$ (Achieved)

2. $2ab^2 + 2a^2b$

$2ab(\quad)$

$2ab(\cancel{2a}b^{\cancel{2}} + \cancel{2}a^{\cancel{2}}\cancel{b})$

$2ab(b + a)$ (Achieved)

PRACTICE

3. $3n^3 - n$

$n(\quad)$

$n(3n^{\cancel{3}(2)} - \cancel{n})$

$n(3n^2 - 1)$ (Achieved)

4. $6x^2 - 18x$

$6x(\quad)$

$6x(\cancel{6}x^{\cancel{2}} - \cancel{18}(3)\cancel{x})$

$6x(x - 3)$ (Achieved)

5. $7x - 49x^2$

$7x(\quad)$

$7x(\cancel{7x} - \cancel{49}(7)x^{\cancel{2}})$

$7x(1 - 7x)$ (Achieved)

6. $8d^7 - 2d^3$

$2d^3(\quad)$

$2d^3(\cancel{8}(4)d^{\cancel{7}(4)} - \cancel{2d}^{\cancel{3}})$

$2d^3(4d^4 - 1)$ (Achieved)

7. $6x^5y^3 - 9x^3y^6$

$3x^3y^3(\quad)$

$3x^3y^3(\cancel{6}(2)x^{\cancel{5}(2)}\cancel{y}^{\cancel{3}} - \cancel{9}(3)\cancel{x}^{\cancel{3}}y^{\cancel{6}(3)})$

$3x^3y^3(2x^2 - 3y^3)$ (Achieved)

8. $3ab^2 + 6b^2$

$3b^2(\quad)$

$3b^2(\cancel{3}a\cancel{b}^{\cancel{2}} + \cancel{6}(2)\cancel{b}^{\cancel{2}})$

$3b^2(a + 2)$ (Achieved)

9. $24g - 18g^3$

$6g(\quad)$

$6g(\cancel{24}(4)\cancel{g} - \cancel{18}(3)g^{\cancel{3}(2)})$

$6g(4 - 3g^2)$ (Achieved)

10. $x^3y^2 + 3xy^4$

$xy^2(\quad)$

$xy^2(x^{\cancel{3}(2)}\cancel{y}^{\cancel{2}} + 3\cancel{x}y^{\cancel{4}(2)})$

$xy^2(x^2 + 3y^2)$ (Achieved)

11. $15a^2b^3 - 10ab^5$

$5ab^3(\quad)$

$5ab^3(\cancel{15}(3)a^{\cancel{2}}\cancel{b}^{\cancel{3}} - \cancel{10}(2)\cancel{a}b^{\cancel{5}(2)})$

$5ab^3(3a - 2b^2)$ (Achieved)

12. $3xy^3 - 4x^3y^2$

$xy^2(\quad)$

$xy^2(3\cancel{x}y^{\cancel{3}} - 4x^{\cancel{3}(2)}\cancel{y}^{\cancel{2}})$

$xy^2(3y - 4x^2)$ (Achieved)

FACTORISING - QUADRATICS

STEPS:	EXAMPLE: $x^2-3x-10$
1. Look for all of the combinations of number that multiply to make the last number **2.** See which of these combinations can add to make the middle number **3.** Write out two sets of brackets with an *x* at the start of each one **4.** Put your two numbers in the brackets with the x	10×1 and 5×2 10 and 1 cannot make -3, 2–5 does make -3 so 2 and -5 are the numbers $(x\quad)(x\quad)$ $(x+2)(x-5)$

For a complete tutorial on this topic visit www.learncoach.co.nz

OLD NCEA QUESTIONS

Factorise the following equations:

1. x^2-2x-3

2. x^2-6x-7

3. x^2-4x-5

4. $x^2+7x+10$

5. $x^2-8x+12$

6. $x^2+7x-60$

PRACTICE QUESTIONS

Factorise the following equations:

7. $x^2-5x-14$

8. $x^2-18x+32$

9. x^2+8x-9

10. x^2-2x-8

11. x^2+7x-8

12. x^2+7x+6

13. x^2-4

14. x^2+2x-8

15. x^2-5x-6

16. $x^2+9x+18$

ANSWERS

NCEA

1. $x^2 - 2x - 3$
$\mathbf{1 \times 3}\ (1 - 3 = -2)$
$(x + 1)(x - 3)$ (Achieved)

2. $x^2 - 6x - 7$
$\mathbf{1 \times 7}\ (1 - 7 = 6)$
$(x + 1)(x - 7)$ (Achieved)

3. $x^2 - 4x - 5$
$\mathbf{1 \times 5}\ (1 - 5 = -4)$
$(x + 1)(x - 5)$ (Achieved)

4. $x^2 + 7x + 10$
$1 \times 10,\ \mathbf{2 \times 5}\ (2 + 5 = 7)$
$(x + 2)(x + 5)$ (Achieved)

5. $x^2 - 8x + 12$
$1 \times 12,\ \mathbf{2 \times 6},\ 3 \times 4\ (-2 - 6 = -8)$
$(x - 2)(x - 6)$
(Achieved)

6. $x^2 + 7x - 60$
$1 \times 60,\ 2 \times 30,\ 3 \times 20,\ 4 \times 15,$
$\mathbf{5 \times 12},\ 6 \times 10\ (12 - 5 = 7)$
$(x + 12)(x - 5)$ (Achieved)

PRACTICE

7. $x^2 - 5x - 14$
$1 \times 14,\ \mathbf{2 \times 7}\ (2 - 7 = -5)$
$(x + 2)(x - 7)$ (Achieved)

8. $x^2 - 18x + 32$
$1 \times 32,\ \mathbf{2 \times 16},\ 4 \times 8\ (-2 - 16 = -18)$
$(x - 2)(x - 16)$ (Achieved)

9. $x^2 + 8x - 9$
$\mathbf{1 \times 9},\ 3 \times 3\ (9 - 1 = 8)$
$(x + 9)(x - 1)$ (Achieved)

10. $x^2 - 2x - 8$
$1 \times 8,\ \mathbf{2 \times 4}\ (2 - 4 = -2)$
$(x + 2)(x - 4)$ (Achieved)

11. $x^2 + 7x - 8$
$\mathbf{1 \times 8},\ 2 \times 4\ (8 - 1 = 7)$
$(x + 8)(x - 1)$ (Achieved)

12. $x^2 + 7x + 6$
$\mathbf{1 \times 6},\ 2 \times 3\ (6 + 1 = 7)$
$(x + 6)(x + 1)$ (Achieved)

13. $x^2 - 4 = x^2 + 0x - 4$
$1 \times 4,\ \mathbf{2 \times 2}\ (2 - 2 = 0)$
$(x + 2)(x - 2)$ (Achieved)

14. $x^2 + 2x - 8$
$1 \times 8,\ \mathbf{2 \times 4}\ (4 - 2 = 2)$
$(x + 4)(x - 2)$ (Achieved)

15. $x^2 - 5x - 6$
$\mathbf{1 \times 6},\ 2 \times 3\ (1 - 6 = -5)$
*It is not -2 and -3 as they do not multiply to -6
$(x + 1)(x - 6)$ (Achieved)

16. $x^2 + 9x + 18$
$1 \times 18,\ 2 \times 9,\ \mathbf{3 \times 6}\ (3 + 6 = 9)$
$(x + 3)(x + 6)$ (Achieved)

SIMPLIFY FRACTIONS WITH QUADRATICS

STEPS:	EXAMPLE: $\frac{x^2+8x+12}{x^2+6x}$
1. Factorise the top	$\frac{(x+2)(x+6)}{x^2+6x}$
2. Factorise the bottom	$\frac{(x+2)(x+6)}{x(x+6)}$
3. Cancel the common factors	$\frac{(x+2)\cancel{(x+6)}}{x\cancel{(x+6)}} = \frac{x+2}{x}$

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OLD NCEA QUESTIONS

Simplify the following fractions:

1. $\frac{x^2-4x-5}{x^2+6x+5}$

2. $\frac{x^2-6x-7}{x^2+5x+4}$

3. $\frac{x^2-2x-3}{x^2-7x+12}$

4. $\frac{x^2+7x+10}{x^2+2x}$

PRACTICE QUESTIONS

Simplify the following fractions:

5. $\frac{x^2-x-6}{x^2+6x+8}$

6. $\frac{x^2-9}{x^2-8x+15}$

7. $\frac{x^2+2x-8}{3x^2+12x}$

8. $\frac{x^2+x-12}{x^2-2x-3}$

9. $\frac{x^2+2x+1}{x^2+5x+4}$

10. $\frac{4x-16}{x^2-2x-8}$

11. $\frac{x^2-14x-32}{2x^2-32x}$

12. $\frac{x^2-x-30}{x^2-14x+48}$

13. $\frac{x^2-4x-5}{x^2+x-30}$

14. $\frac{x^2-7x}{x^2-3x-28}$

ANSWERS

NCEA

1. $\dfrac{x^2-4x-5}{x^2+6x+5}$

$\dfrac{(x+1)(x-5)}{x^2+6x+5}$

$\dfrac{(x+1)(x-5)}{(x+1)(x+5)}$

$\dfrac{x-5}{x+5}$ (Merit)

2. $\dfrac{x^2-6x-7}{x^2+5x+4}$

$\dfrac{(x+1)(x-7)}{x^2+5x+4}$

$\dfrac{(x+1)(x-7)}{(x+1)(x+4)}$

$\dfrac{x-7}{x+4}$ (Merit)

3. $\dfrac{x^2-2x-3}{x^2-7x+12}$

$\dfrac{(x+1)(x-3)}{x^2-7x+12}$

$\dfrac{(x+1)(x-3)}{(x-3)(x-4)}$

$\dfrac{x+1}{x-4}$ (Merit)

4. $\dfrac{x^2+7x+10}{x^2+2x}$

$\dfrac{(x+2)(x+5)}{x^2+2x}$

$\dfrac{(x+2)(x+5)}{x(x+2)}$

$\dfrac{x+5}{x}$ (Merit)

PRACTICE

5. $\dfrac{x^2-x-6}{x^2+6x+8}$

$\dfrac{(x+2)(x-3)}{x^2+6x+8}$

$\dfrac{(x+2)(x-3)}{(x+2)(x+4)}$

$\dfrac{x-3}{x+4}$ (Merit)

6. $\dfrac{x^2-9}{x^2-8x+15}$

$\dfrac{(x+3)(x-3)}{x^2-8x+15}$

$\dfrac{(x+3)(x-3)}{(x-3)(x-5)}$

$\dfrac{x+3}{x-5}$ (Merit)

7. $\dfrac{x^2+2x-8}{3x^2+12x}$

$\dfrac{(x-2)(x+4)}{3x^2+12x}$

$\dfrac{(x-2)(x+4)}{3x(x+4)}$

$\dfrac{x-2}{3x}$ (Merit)

8. $\dfrac{x^2+x-12}{x^2-2x-3}$

$\dfrac{(x-3)(x+4)}{x^2-2x-3}$

$\dfrac{(x-3)(x+4)}{(x+1)(x-3)}$

$\dfrac{x+4}{x+1}$ (Merit)

9. $\dfrac{x^2+2x+1}{x^2+5x+4}$

$\dfrac{(x+1)^2}{x^2+5x+4}$

$\dfrac{(x+1)^2}{(x+1)(x+4)}$

$\dfrac{x+1}{x+4}$ (Merit)

10. $\dfrac{4x-16}{x^2-2x-8}$

$\dfrac{4(x-4)}{x^2-2x-8}$

$\dfrac{4(x-4)}{(x+2)(x-4)}$

$\dfrac{4}{x+2}$ (Merit)

11. $\dfrac{x^2-14x-32}{2x^2-32x}$

$\dfrac{(x-16)(x+2)}{2x^2-32x}$

$\dfrac{(x-16)(x+2)}{2x(x-16)}$

$\dfrac{x+2}{2x}$ (Merit)

12. $\dfrac{x^2-x-30}{x^2-14x+48}$

$\dfrac{(x+5)(x-6)}{x^2-14x+48}$

$\dfrac{(x+5)(x-6)}{(x-6)(x-8)}$

$\dfrac{x+5}{x-8}$ (Merit)

13. $\dfrac{x^2-4x-5}{x^2+x-30}$

$\dfrac{(x+1)(x-5)}{x^2+x-30}$

$\dfrac{(x+1)(x-5)}{(x+6)(x-5)}$

$\dfrac{x+1}{x+6}$ (Merit)

14. $\dfrac{x^2-7x}{x^2-3x-28}$

$\dfrac{x(x-7)}{x^2-3x-28}$

$\dfrac{x(x-7)}{(x+4)(x-7)}$

$\dfrac{x}{x+4}$ (Merit)

SOLVING LINEAR EQUATIONS

STEPS:	EXAMPLE: $5x+7=x-2$
These are all of the steps, you will ever need to perform all in one question. So skip the steps which are not applicable:	
1. Multiply away any fractions	$(none)-skip$
2. Expand any brackets	$(none)-skip$
3. Put x's on the same side	$5x+7-\boldsymbol{x}=x-2-\boldsymbol{x}$
4. Put numbers on the opposite side to the x's	$5x+7-\boldsymbol{x}-\mathbf{7}=-2-\mathbf{7}$
5. Simplify	$4x=-9$
6. Divide by the last number	$x=\frac{-9}{4}$

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OLD NCEA QUESTIONS

Solve the following equations:

1. $5x-2=7x+10$

2. $2(x-7)=20$

3. $\frac{2-5x}{4}=3$

4. $\frac{3-4x}{5}=7$

5. $6x-2=8x+6$

6. $2(x-2)=24$

PRACTICE QUESTIONS

Solve the following equations:

7. $8x-9=27$

8. $6x+8=x-3$

9. $\frac{5x}{3}-7=13$

10. $\frac{7x+2}{8}=9$

11. $9x-1=4x+7$

12. $4x+6=x-9$

13. $\frac{4x}{5}=\frac{13}{2}$

14. $\frac{4x}{3}-2=14$

15. $6(1-e)=7$

16. $7x-3=3x+9$

ANSWERS

NCEA

1. $5x-2=7x+10$
$5x-2-\mathbf{7x}=7x+10-\mathbf{7x}$
$-2x-2+\mathbf{2}=10+\mathbf{2}$
$-2x=12$
$x=\dfrac{12}{-2}=-6$ (Achieved)

2. $2(x-7)=20$
$2\times x-2\times 7=20$
$2x-14+\mathbf{14}=20+\mathbf{14}$
$2x=34$
$x=\dfrac{34}{2}=17$ (Achieved)

3. $\dfrac{2-5x}{4}=3$
$2-5x=3\times 4$
$2-5x-\mathbf{2}=12-\mathbf{2}$
$-5x=10$
$x=\dfrac{10}{-5}=-2$ (Achieved)

4. $\dfrac{3-4x}{5}=7$
$3-4x=7\times 5$
$3-4x-\mathbf{3}=35-\mathbf{3}$
$-4x=32$
$x=\dfrac{32}{-4}=-8$ (Achieved)

5. $6x-2=8x+6$
$6x-2-\mathbf{8x}=8x+6-\mathbf{8x}$
$-2x-2+\mathbf{2}=6+\mathbf{2}$
$-2x=8$
$x=\dfrac{8}{-2}=-4$ (Achieved)

6. $2(x-2)=24$
$2\times x-2\times 2=24$
$2x-4+\mathbf{4}=24+\mathbf{4}$
$2x=28$
$x=\dfrac{28}{2}=14$ (Achieved)

PRACTICE

7. $8x-9=27$
$8x-9+\mathbf{9}=27+\mathbf{9}$
$8x=36$
$x=\dfrac{36}{8}=\dfrac{9}{2}$ (Achieved)

8. $6x+8=x-3$
$6x+8-\mathbf{x}=x-3-\mathbf{x}$
$5x+8-\mathbf{8}=-3-\mathbf{8}$
$5x=-11$
$x=\dfrac{-11}{5}$ (Achieved)

9. $\dfrac{5x}{3}-7=13$
$5x-7\times 3=13\times 3$
$5x-21+\mathbf{21}=39+\mathbf{21}$
$5x=60$
$x=\dfrac{60}{5}=12$ (Achieved)

10. $\dfrac{7x+2}{8}=9$
$7x+2=9\times 8$
$7x+2-\mathbf{2}=72-\mathbf{2}$
$7x=70$
$x=\dfrac{70}{7}=10$ (Achieved)

11. $9x-1=4x+7$
$9x-1-\mathbf{4x}=4x+7-\mathbf{4x}$
$5x-1+\mathbf{1}=7+\mathbf{1}$
$5x=8$
$x=\dfrac{8}{5}$ (Achieved)

12. $4x+6=x-9$
$4x+6-\mathbf{x}=x-9-\mathbf{x}$
$3x+6-\mathbf{6}=-9-\mathbf{6}$
$3x=-15$
$x=\dfrac{-15}{3}=-5$ (Achieved)

13. $\dfrac{4x}{5}=\dfrac{13}{2}$
$4x\times 2=5\times 13$
$8x=65$
$x=\dfrac{65}{8}$ (Achieved)

14. $\dfrac{4x}{3}-2=14$
$4x-2\times 3=14\times 3$
$4x-6+\mathbf{6}=42+\mathbf{6}$
$4x=48$
$x=\dfrac{48}{4}=12$ (Achieved)

15. $6(1-e)=7$
$6-6e=7$
$6-6e-\mathbf{6}=7-\mathbf{6}$
$-6e=1$
$e=-\dfrac{1}{6}$ (Achieved)

16. $7x-3=3x+9$
$7x-3-\mathbf{3x}=3x+9-\mathbf{3x}$
$4x-3+\mathbf{3}=9+\mathbf{3}$
$4x=12$
$x=\dfrac{12}{4}=3$ (Achieved)

SOLVING QUADRATICS

STEPS:	EXAMPLE: $x^2 - 3x = 10$
1. Make the equation = 0	$x^2 - 3x - 10 = 0$
2. Factorise (if unsure see prev. steps)	$(x + 2)(x - 5) = 0$
3. Make each bracket = 0	$x + 2 = 0$ and $x - 5 = 0$
4. Solve for x (twice)	$x = -2$ and $x = 5$

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OLD NCEA QUESTIONS

Solve the following equations:

1. $x^2 - 2x - 3 = 0$

2. $x^2 - 6x - 7 = 0$

3. $x^2 - 4x - 5 = 0$

4. $x^2 - 7x + 6 = 0$

PRACTICE QUESTIONS

Solve the following equations:

5. $(x - 8)(x + 5) = 0$

6. $4x(x - 8) = 0$

7. $x^2 - 4x - 12 = 0$

8. $x^2 - 13x = -42$

9. $5x(2x - 7) = 0$

10. $(7x + 9)(2x + 7) = 0$

11. $x^2 + 12x + 35 = 0$

12. $x^2 - 8x + 7 = 0$

13. $x^2 - 52x + 100 = 0$

14. $x^2 + 13x + 78 = 36$

ANSWERS

NCEA

1. $x^2 - 2x - 3 = 0$
$(x+1)(x-3) = 0$
$x+1=0$ and $x-3=0$
$x = -1$ and $x = 3$ (Merit)

2. $x^2 - 6x - 7 = 0$
$(x-7)(x+1) = 0$
$x-7=0$ and $x+1=0$
$x = 7$ and $x = -1$ (Merit)

3. $x^2 - 4x - 5 = 0$
$(x-5)(x+1) = 0$
$x-5=0$ and $x+1=0$
$x = 5$ and $x = -1$ (Merit)

4. $x^2 - 7x + 6 = 0$
$(x-6)(x-1) = 0$
$x-6=0$ and $x-1=0$
$x = 6$ and $x = 1$ (Merit)

PRACTICE

5. $(x-8)(x+5) = 0$
$x-8=0$ and $x+5=0$
$x = 8$ and $x = -5$ (Merit)

6. $4x(x-8) = 0$
$4x=0$ and $x-8=0$
$x = 0$ and $x = 8$ (Merit)

7. $x^2 - 4x - 12 = 0$
$(x-6)(x+2) = 0$
$x-6=0$ and $x+2=0$
$x = 6$ and $x = -2$ (Merit)

8. $x^2 - 13x = -42$
$x^2 - 13x + 42 = 0$
$(x-6)(x-7) = 0$
$x-6=0$ and $x-7=0$
$x = 6$ and $x = 7$ (Merit)

9. $5x(2x-7) = 0$
$5x=0$ and $2x-7=0$
$x = 0$ and $x = \frac{7}{2}$ (Merit)

10. $(7x+9)(2x+7) = 0$
$7x+9=0$ and $2x+7=0$
$x = -\frac{9}{7}$ and $x = -\frac{7}{2}$ (Merit)

11. $x^2 + 12x + 35 = 0$
$(x+5)(x+7) = 0$
$x+5=0$ and $x+7=0$
$x = -5$ and $x = -7$ (Merit)

12. $x^2 - 8x + 7 = 0$
$(x-7)(x-1) = 0$
$x-7=0$ and $x-1=0$
$x = 7$ and $x = 1$ (Merit)

13. $x^2 - 52x + 100 = 0$
$(x-50)(x-2) = 0$
$x-50=0$ and $x-2=0$
$x = 50$ and $x = 2$ (Merit)

14. $x^2 + 13x + 78 = 36$
$x^2 + 13x + 42 = 0$
$(x+6)(x+7) = 0$
$x+6=0$ and $x+7=0$
$x = -6$ and $x = -7$ (Merit)

SOLVING FRACTIONS WITH QUADRATICS

STEPS:	EXAMPLE: $\frac{x^2+8x+12}{x^2+6x}=2$
1. Factorise top and bottom	$\frac{(x+2)(x+6)}{x(x+6)}=2$
2. Cancel common factors	$\frac{x+2}{x}=2$
3. Multiply both sides by the bottom	$x+2=2x$
4. Solve as a linear equation	$x+2-x=2x-x$ $2=x$

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OLD NCEA QUESTIONS

Solve the following fractions:

1. $\frac{x^2-4x-5}{x^2+6x+5}=2$

2. $\frac{x^2-6x-7}{x^2+5x+4}=2$

3. $\frac{x^2-2x-3}{x^2-7x+12}=2$

PRACTICE QUESTIONS

Solve the following fractions:

4. $\frac{x^2+x-12}{6x-18}=1$

5. $\frac{x^2+2x}{x^2-x-6}=2$

6. $\frac{x^2+x-42}{x^2-14x+48}=2$

7. $\frac{x^2+9x+20}{x^2-25}=10$

8. $\frac{x^2-6x-7}{x^2-7x}=5$

9. $\frac{x^2-x-12}{x^2-2x-8}=2$

10. $\frac{x^2+3x-4}{x^2+5x+4}=3$

11. $\frac{x^2+x-6}{x^2-6x+8}=3$

12. $\frac{x^2+2x-3}{x^2-3x+18}=4$

13. $\frac{x^2-7x+10}{x^2+x-6}=3$

ANSWERS

NCEA

1. $\frac{x^2-4x-5}{x^2+6x+5}=2$

$\frac{(x+1)(x-5)}{(x+1)(x+5)}=2$

$\frac{x-5}{x+5}=2$

$x-5=2(x+5)$

$x-5=2x+10$

$-15=x$ (Excellence)

2. $\frac{x^2-6x-7}{x^2+5x+4}=2$

$\frac{(x+1)(x-7)}{(x+1)(x+4)}=2$

$\frac{x-7}{x+4}=2$

$x-7=2(x+4)$

$x-7=2x+8$

$-15=x$ (Excellence)

3. $\frac{x^2-2x-3}{x^2-7x+12}=2$

$\frac{(x+1)(x-3)}{(x-3)(x-4)}=2$

$\frac{x+1}{x-4}=2$

$x+1=2(x-4)$

$x+1=2x-8$

$9=x$ (Excellence)

PRACTICE

4. $\frac{x^2+x-12}{6x-18}=1$

$\frac{(x+4)(x-3)}{6(x-3)}=1$

$\frac{x+4}{6}=1$

$x+4=6$

$x=2$ (Excellence)

5. $\frac{x^2+2x}{x^2-x-6}=2$

$\frac{x(x+2)}{(x+2)(x-3)}=2$

$\frac{x}{x-3}=2$

$x=2(x-3)$

$x=2x-6$

$6=x$ (Excellence)

6. $\frac{x^2+x-42}{x^2-14x+48}=2$

$\frac{(x+7)(x-6)}{(x-6)(x-8)}=2$

$\frac{x+7}{x-8}=2$

$x+7=2(x-8)$

$x+7=2x-16$

$23=x$ (Excellence)

7. $\frac{x^2+9x+20}{x^2-25}=10$

$\frac{(x+5)(x+4)}{(x+5)(x-5)}=10$

$\frac{x+4}{x-5}=10$

$x+4=10(x-5)$

$54=9x$

$6=x$ (Excellence)

8. $\frac{x^2-6x-7}{x^2-7x}=5$

$\frac{(x+1)(x-7)}{x(x-7)}=5$

$\frac{x+1}{x}=5$

$x+1=5x$

$1=4x$

$\frac{1}{4}=x$ (Excellence)

9. $\frac{x^2-x-12}{x^2-2x-8}=2$

$\frac{(x+3)(x-4)}{(x+2)(x-4)}=2$

$\frac{x+3}{x+2}=2$

$x+3=2(x+2)$

$x+3=2x+4$

$-1=x$ (Excellence)

10. $\frac{x^2+3x-4}{x^2+5x+4}=3$

$\frac{(x-1)(x+4)}{(x+4)(x+1)}=3$

$\frac{x-1}{x+1}=3$

$x-1=3(x+1)$

$x-1=3x+3$

$-4=2x$

$-2=x$ (Excellence)

11. $\frac{x^2+x-6}{x^2-6x+8}=3$

$\frac{(x+3)(x-2)}{(x-2)(x-4)}=3$

$\frac{x+3}{x-4}=3$

$x+3=3(x-4)$

$x+3=3x-12$

$15=2x$

$\frac{15}{2}=x$ (Excellence)

12. $\frac{x^2+2x-3}{x^2-3x+18}=4$

$\frac{(x+3)(x-1)}{(x+3)(x-6)}=4$

$\frac{x-1}{x-6}=4$

$x-1=4(x-6)$

$x-1=4x-24$

$23=3x$

$\frac{23}{3}=x$ (Excellence)

13. $\frac{x^2-7x+10}{x^2+x-6}=3$

$\frac{(x-5)(x-2)}{(x+3)(x-2)}=3$

$\frac{x-5}{x+3}=3$

$x-5=3(x+3)$

$x-5=3x+9$

$-14=2x$

$-7=x$ (Excellence)

SOLVING EXPONENTS

STEPS:	EXAMPLE: $(y^2)^2 = 81$
1. Simplify with the following rules: • If the base is the same add the powers. e.g. $x^m x^n = x^{m+n}$ • If the exponent is raised to a power multiply the exponent and power. e.g. $(x^m)^n = x^{m \times n}$	$y^4 = 81$ $y \times y \times y \times y = 81$
2. Trial with 2	$2 \times 2 \times 2 \times 2 \neq 81$ **No**
3. Trial with 3	$3 \times 3 \times 3 \times 3 = 81$ **Yes!**
4. Trial with 4 Etc.	Unnecessary.
5. If the power is even numbered you have both +ve and -ve answers	$y = +3$ and -3 as there is an even power (4)

For a complete tutorial on this topic visit www.learncoach.co.nz

OLD NCEA QUESTIONS

Solve the following exponents:

1. $(c^2)^3 = 64$

2. $x^4 = 81$

3. $(x^3)^2 = 64$

PRACTICE QUESTIONS

Solve the following exponents:

4. $x^3 = 64$

5. $x^5 = 32$

6. $(x^4)(x^2) = 64$

7. $(x^2)^2 = 81$

8. $x^3 = 125$

9. $x^3 = 27$

10. $x(x^5) = 64$

11. $(x^3)(x^2) = 32$

ANSWERS

NCEA

1. $(c^2)^3 = 64$
$c^6 = 64$
$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$ **Yes**
$c = 2$ and -2 (Excellence)

2. $x^4 = 81$
$2 \times 2 \times 2 \times 2 \neq 81$ **No**
$3 \times 3 \times 3 \times 3 = 81$ **Yes**
$x = 3$ and -3 (Excellence)

3. $(x^3)^2 = 64$
$x^6 = 64$
$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$ **Yes**
$x = 2$ and -2 (Excellence)

PRACTICE

4. $x^3 = 64$
$2 \times 2 \times 2 \neq 64$ **No**
$3 \times 3 \times 3 \neq 64$ **No**
$4 \times 4 \times 4 = 64$ **Yes**
$x = 4$ (Merit)

5. $x^5 = 32$
$2 \times 2 \times 2 \times 2 \times 2 = 32$ **Yes**
$x = 2$ (Merit)

6. $(x^4)(x^2) = 64$
$x^6 = 64$
$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$ **Yes**
$x = 2$ and -2 (Excellence)

7. $(x^2)^2 = 81$
$x^4 = 81$
$2 \times 2 \times 2 \times 2 \neq 81$ **No**
$3 \times 3 \times 3 \times 3 = 81$ **Yes**
$x = 3$ and -3 (Excellence)

8. $x^3 = 125$
$2 \times 2 \times 2 \neq 125$ **No**
$3 \times 3 \times 3 \neq 125$ **No**
$4 \times 4 \times 4 \neq 125$ **No**
$5 \times 5 \times 5 = 125$ **Yes**
$x = 5$ (Merit)

9. $x^3 = 27$
$2 \times 2 \times 2 \neq 27$ **No**
$3 \times 3 \times 3 = 27$ **Yes**
$x = 3$ (Merit)

10. $x(x^5) = 64$
$x^6 = 64$
$2 \times 2 \times 2 \times 2 \times 2 \times 2$ **Yes**
$x = 2$ and -2 (Excellence)

11. $(x^3)(x^2) = 32$
$x^5 = 32$
$2 \times 2 \times 2 \times 2 \times 2 = 32$ **Yes**
$x = 2$ (Merit)

SOLVING INEQUALITIES

STEPS:	EXAMPLE: $\frac{2(x-2)}{4} > 3$
1. Multiply away any fractions	$2(x-2) > 3 \times 4$
2. Expand any brackets	$2x - 4 > 12$
3. Put x's on the same side	$(none) - skip$
4. Put numbers on the opposite side	$2x - 4 + \mathbf{4} > 12 + \mathbf{4}$
5. Simplify	$2x > 16$
6. Divide by the last number (**Remember** if dividing by a negative number reverse the sign)	$x > 8$

For a complete tutorial on this topic visit www.learncoach.co.nz

OLD NCEA QUESTIONS

Solve the following inequalities:

1. $\frac{4-2x}{3} > 8$

2. $\frac{2-5x}{4} > 3$

3. $\frac{3-4x}{5} > 7$

PRACTICE QUESTIONS

Solve the following inequalities:

4. $\frac{3x-1}{4} > 5$

5. $\frac{4(x+6)}{5} < 2x$

6. $3x + 7 > 4x$

7. $\frac{2-x}{4} > 3$

8. $6(x-3) < 2$

9. $14 - 4x < 6$

10. $\frac{5x-8}{3} > 7x$

11. $9(3-x) < 4(2-x) - 1$

12. $\frac{2(3-2x)}{3} > 6$

13. $5x + 7 < 2x + 3$

ANSWERS

NCEA

1. $\frac{4-2x}{3} > 8$

$4-2x > 8\times 3$

$4-2x-\mathbf{4} > 24-\mathbf{4}$

$-2x > 20$

$x < -10$ (Merit)

2. $\frac{2-5x}{4} > 3$

$2-5x > 3\times 4$

$2-5x-\mathbf{2} > 12-\mathbf{2}$

$-5x > 10$

$x < -2$ (Merit)

3. $\frac{3-4x}{5} > 7$

$3-4x > 7\times 5$

$3-4x-\mathbf{3} > 35-\mathbf{3}$

$-4x > 32$

$x < -8$ (Merit)

PRACTICE

4. $\frac{3x-1}{4} > 5$

$3x-1 > 5\times 4$

$3x-1+\mathbf{1} > 20+\mathbf{1}$

$3x > 21$

$x > 7$ (Merit)

5. $\frac{4(x+6)}{5} < 2x$

$4\times x+4\times 6 < 5\times 2x$

$4x+24-\mathbf{4x} < 10x-\mathbf{4x}$

$24 < 6x$

$4 < x$ (Merit)

6. $3x+7 > 4x$

$3x+7-\mathbf{3x} > 4x-\mathbf{3x}$

$7 > x$ (Merit)

7. $\frac{2-x}{4} > 3$

$2-x > 3\times 4$

$2-x-\mathbf{2} > 12-\mathbf{2}$

$-x > 10$

$x < -10$ (Merit)

8. $6(x-3) < 2$

$6\times x-6\times 3 < 2$

$6x-18+\mathbf{18} < 2+\mathbf{18}$

$6x < 20$

$x < \frac{20}{6}$ or $\frac{10}{3}$ (Merit)

9. $14-4x < 6$

$14-4x-\mathbf{14} < 6-\mathbf{14}$

$-4x < -8$

$x > 2$ (Merit)

10. $\frac{5x-8}{3} > 7x$

$5x-8 > 3\times 7x$

$5x-8-\mathbf{5x} > 21x-\mathbf{5x}$

$-8 > 16x$

$-\frac{1}{2} > x$ (Merit)

11. $9(3-x) < 4(2-x)-1$

$9\times 3-9\times x < 4\times 2-4\times x-1$

$27-9x+\mathbf{4x}-\mathbf{27} < 7-4x+\mathbf{4x}-\mathbf{27}$

$-5x < -20$

$x > 4$ (Merit)

12. $\frac{2(3-2x)}{3} > 6$

$2\times 3-2\times 2x > 6\times 3$

$6-4x-\mathbf{6} > 18-\mathbf{6}$

$-4x > 12$

$x < -3$ (Merit)

13. $5x+7 < 2x+3$

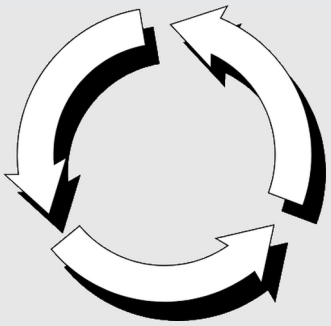
$5x+7-\mathbf{2x}-\mathbf{7} < 2x+3-\mathbf{2x}-\mathbf{7}$

$3x < -4$

$x < \frac{-4}{3}$ (Merit)

Study Tip:

Repetition



*To **burn information into your brain**:*
***Don't** study the same topic for a long period of time*
***Do** learn the information then go over it again:*

- *Within **24** hours (retention goes from **20%** to **80%**)*
- *Again after a week (retention goes up to **90%**)*
- *Again after a **month** (**long term** retention)*

SUBSTITUTION AND REARRANGING

STEPS:	EXAMPLE: $T = 2\pi\sqrt{\frac{l}{g}}$
Substitution: If l is 90 and g is 10. Give T in terms of π. **1.** Put the numbers in place of the letters	$T = 2\pi\sqrt{\frac{90}{10}}$
2. Simplify or solve the term	$T = 2\pi\sqrt{9}$ $T = 2\pi \times 3$ $T = 6\pi$
Rearranging: Make g the subject. **1.** Rearrange to make sure that the subject is a numerator (on the top) not a denominator (on the bottom)	$\frac{T}{2p} = \sqrt{\frac{l}{g}}$ $\left(\frac{T}{2p}\right)^2 = \frac{l}{g}$ $g\left(\frac{T}{2p}\right)^2 = l$
2. Get rid of all other symbols/numbers around the subject	$g = \frac{l}{\left(\frac{T}{2\pi}\right)^2}$

For a complete tutorial on this topic visit www.learncoach.co.nz

OLD NCEA QUESTIONS

1. Sara found the following equation:

$$A = \pi\sqrt{\frac{w}{g}}$$

a. If W = 25 and g = 16, find A in terms of π.

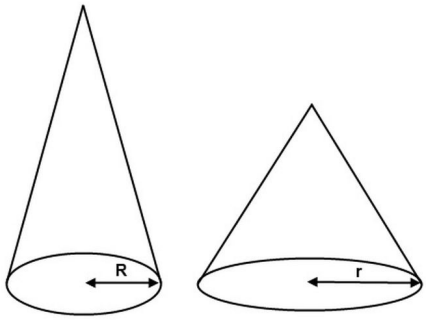
b. Sara now wants to use the formula to find W in a different situation.
Rewrite the formula with W as the subject.

2. The formula for the volume of a cone is $v = \frac{\pi}{3}r^2h$

where r is the radius and h is the height of the cone.

a. Write the formula for the radius, r, of the cone in terms of V, h, and π.

b. Max has two cones that have the same volume. One cone is twice the height of the other. Give an expression for the radius, r, of the shorter cone in terms of R, the radius of the taller cone. Give your answer in the simplest form.



3. Mele is exploring the sequence of numbers given by the rule:

$3n^2 - 2n + 1$

Give the rule for finding the difference between any two consecutive terms from the sequence $3n^2 - 2n + 1$ in its simplest form.

Hint: consecutive terms follow each other, e.g. the 5th and 6th terms or the 17th and 18th terms or the n^{th} and $(n+1)^{th}$ terms.

You must show your working.

PRACTICE QUESTIONS

4. A square tile is surrounded by a 4 cm wide border. The area of the tile is x^2. The area of the border is $(x+8)^2 - x^2$. If the border covers an area of 640 cm^2, what is the tile length in centimetres?

5. A formula for the perimeter, G, of a rectangle is $G = 2j + 2s$ where j is the length of the base of the rectangle and s is the height of the rectangle. Make j the subject of this formula.

6. Make h the subject of $4l = 7(h-10)$

7. Rearrange $2K + 3H = 450$ to make H the subject.

8. Jeans sizes are even numbers. The size J can be calculated from the waist measurement m using the formula: $J = \frac{4m - 90}{6}$ where m is in centimetres.

a. Rearrange to make m the subject of the formula.

b. What size jeans should Henry order, if his waist measurement is 71cm?

9. Posting parcels is expensive. It depends both on the weight of the parcel and the distance it has to travel. The rates are $3 per kg and 15 cents per km. The cost (P) of sending a parcel can be calculated by the formula $P = 3w + 0.15d$ where w is the weight in kg and d is the distance in km. The distance from Auckland to Invercargill is 1572km. Find the cost of sending a parcel weighing 8kg from Auckland to Invercargill.

10. In a trampoline competition the score (T) for a trick is calculated using the formula $T = 1.5HF$ where F is the difficulty of the trick and H is the height of the jump. John completes a trick of difficulty 8. The height of his jump was 1.5m. Calculate the score for John.

11. May prints butterflies on different shaped pillows. Her rule for calculating the total number of butterflies (B) she prints on the pillow is $B = \frac{n(n+1)}{2}$ where n is the number of edges on the pillow. Find the total number of butterflies she prints on a pillow with 10 edges.

Study Tip:

Retrieving Information

***Don't** just put information **into** your brain.*
***Do** practice getting the information **out** of your brain.*
***Retrieving** the stored information in your brain by answering lots of questions is **more effective** than reading or taking notes and makes us stronger in exams.*

ANSWERS

NCEA

1.

a. $A = \pi\sqrt{\frac{w}{g}} = \pi\sqrt{\frac{25}{16}}$

$A = \pi\frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}\pi$ (Merit)

b. $A = \pi\sqrt{\frac{w}{g}} \quad \rightarrow \quad \frac{A}{\pi} = \sqrt{\frac{w}{g}}$

$\left(\frac{A}{\pi}\right)^2 = \frac{w}{g} \quad \rightarrow \quad g\left(\frac{A}{\pi}\right)^2 = w$

(Merit)

2.

a. $V = \frac{\pi}{3} / 3r^2h$

$\frac{3V}{\pi h} = r^2$

$r = \sqrt{\frac{3V}{\pi h}}$ (Merit)

b. Let H be the height of the cone with radius R and let h be the height of the cone with radius r. Therefore $H = 2h$. Since the cones have the same volume:

$\frac{\pi}{3}r^2h = \frac{\pi}{3}R^2H$

$r^2h = R^2H$

$r^2 = \frac{R^2H}{h} = \frac{R^2 2h}{h} = 2R^2$

$r = \sqrt{2R^2} = \sqrt{2}R$ (Excellence)

3. The nth term of the sequence is $3n^2 - 2n + 1$ the (n+1)th term of the sequence is $3(n+1)^2 - 2(n+1) + 1$ which simplifies to:

$3\left(n^2 + 2n + 1\right) - 2n - 2 + 1$

$= 3n^2 + 6n + 3 - 2n - 1$

$= 3n^2 + 4n + 2$

so the difference between consecutive terms is:

$3n^2 + 4n + 2 - \left(3n^2 - 2n + 1\right)$

$= 3n^2 + 4n + 2 - 3n^2 + 2n - 1$

$= 6n + 1$ (Excellence)

PRACTICE

4. $(x+8)^2 - x^2 = 640$

$x^2 + 16x + 64 - x^2 = 640$

$16x + 64 = 640$

$16x = 640 - 64$

$x = \frac{576}{16} = 36 \text{ cm}$ (Merit)

5. $G = 2j + 2s$

$2j = G - 2s$

$j = \frac{G}{2} - s$ (Merit)

6. $4l = 7(h - 10)$

$4l = 7h - 70$

$7h = 4l + 70$

$h = \frac{4}{7}l + 10$ (Merit)

7. $2K + 3H = 450$

$3H = 450 - 2K$

$H = 150 - \frac{2}{3}K$ (Merit)

8.

a. $J = \frac{4m - 90}{6}$

$6J = 4m - 90$

$4m = 6J + 90$

$m = \frac{3}{2}J + \frac{45}{2}$ (Merit)

b. $J = \frac{4m - 90}{6}$

$J = \frac{4 \times 71 - 90}{6} = 32.33$

The nearest even number size is 32.

(Merit)

9. $p = 3w + 0.15d$

$p = 3 \times 8 + 0.15 \times 1572$

$p = \$259.80$ (Merit)

10. $T = 1.5HF$

$T = 1.5 \times 1.5 \times 8$

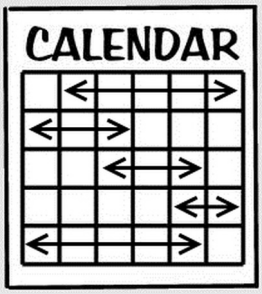
$T = 18$ (Merit)

11. $B = \frac{n(n+1)}{2}$

$B = \frac{10(10+1)}{2} = 55$ (Merit)

Study Tip:

Time Management



*This is the **key** to **successful** study.*
You need to:

- *Decide how to **best use** the hours you have each day, week, and term*
- *Schedule **study** times and **free** time!*
- ***Stick** to your schedule*
- *After each study session, write down the next session time and what you plan to study*

SIMULTANEOUS EQUATIONS

STEPS:	EXAMPLE: $\begin{matrix}4x+3y=46\\ x+3y=16\end{matrix}$
1. Write out the two equations if they haven't been given	$4x+3y=46$ $x+3y=16$
2. Rearrange one equation so that a variable is the subject	$x=16-3y$
3. Substitute into the other equation and solve for one variable	$4(16-3y)+3y=46$ $64-12y+3y=46$ $-9y=-18$ $y=2$
4. Substitute answer into rearranged equation and solve for other variable	$x=16-3y$ $x=16-3\times 2$ $x=10$

For a complete tutorial on this topic visit www.learncoach.co.nz

OLD NCEA QUESTIONS

1. Ari spent \$45 buying some CDs in a sale. He bought R Rock CDs and B Blues CDs.
Ari writes an equation for the amount he spent as: $2R+B=45$.
Ari bought four times as many Rock CDs as Blues CDs.
How many blues CDs did he buy all together?

2. A milk drink costs \$1.50 more than a fruit drink.
5 fruit drinks and 4 milk drinks cost a total of \$24
What is the cost of one milk drink?
You must show at least one equation that can be used in solving this problem.

3. Scenic School is using two vans to take a group of students on a field trip.
If two students move from van A to van B, then the two vans would have the same number of students in each, but if two students moved from van B to van A, then van B would have half the number of students that were then in van A.
Use the information above to find the total number of students on the field trip.
In your answer, you must give at least ONE equation that you would use to solve the problem.

4. A drink costs \$2.50 more than a packet of chips.
2 drinks and 4 packets of chips cost a total of \$17.
What is the cost of 1 drink?
You must show at least one equation that can be used in solving this problem.

PRACTICE QUESTIONS

5. 370 people paid a total of $2330 to watch the Cirque de Soleil. Adults pay $8 entry, while children pay less. These two simultaneous equations can be used to find how many adults paid to watch the circus.

$x + y = 370$
$8x + 5y = 2330$

How many paying adults attended the circus?

6. It costs each adult $12 and each child $5 to get into a soccer game. An $80 group pass to the soccer game is available. This lets in a group of up to 8 people into the game. The group pass is not always the cheapest option. If a group of 8 people is to save money with the group pass, what is the minimum number of adults required?

7. Solve this pair of simultaneous equations for x and y.

$x + y = 10$
$7x + 8y = 200$

8. Fiona bought tickets to events at the Olympic Games. She paid $1095 for 15 tickets. The tickets for the cycling events cost $65 and the tickets for the track and field events cost $85. Solve the simultaneous equations to find the number of tickets Fiona bought to each event.

$65c + 85t = 1095$
$c + t = 15$

9. In a sale at a market, all the seconds fruit were one price and all the export grade fruit was another price. Zachariah bought 2 cartons of seconds fruit and four of export grade fruit and paid in total $39.70. Kirsty bought 3 cartons of seconds fruit and one of export quality and paid $39.80. Be aware the seconds cartons are much larger and therefore more expensive. Solve the following simultaneous equations to calculate the sale price of one export carton.

$2s + 4x = 39.70$
$3s + x = 39.80$

10. Daisy was a flower expert and had been studying roses. She discovered a new species early in 2005 and another later in the year. The trend line for each of her discoveries are: $y = 40x + 175$ and $2y = 85x + 290$. Y is the number of thorns on their stalks and x is the number of years since 1985. Predict the year in which the roses will both have the same number of thorns.

11. Blue and Tasty cheeses are blended together to make new cheeses. Blue cheese has 20% fat and Tasty cheese, 35%. One of the blends of the cheeses is called Tasty Blue Cheese. The Tasty Blue cheese is to have a fat content of 30%. 800 grams of the new blend is to be made in trial. Let B = mass of Blue cheese and T = mass of Tasty cheese in grams.

$20B + 35T = 24000$
$B + T = 800$

Solve the simultaneous equations to find the mass of Tasty and Blue required. Show all your working.

12. The total annual health bill of the residents of Sloth Avenue is $480000. The health costs per adult, H, are $1500 per year more than for a child, C.

$80H + 64C = 480000$
$H = C + 1500$

Solve the simultaneous equations to find the yearly health costs for a child.

Study Tip:

Quizzing

*Being **quizzed** by a **person** is **easier** and more **fun** than learning from a textbook.*
*But remember, if you are doing it with a friend, **stay on task**!*

ANSWERS

NCEA

1. $R = 4B$ and $2R + B = 45$
Sub: $2(4B) + B = 45$
$9B = 45$
$B = 5$
He bought 5 Blues CD's. (Excellence)

2. m = milk drink and f = fruit drink
$m = f + 1.5$ and $5f + 4m = 24$
sub: $5f + 4(f + 1.5) = 24$
$5f + 4f + 6 = 24$
$9f = 18$
$f = \$2$
$m = 2 + 1.5 = \$3.50$ (Excellence)

3. $A - 2 = B + 2$ and $B - 2 = \frac{1}{2}(A + 2)$
or $2(B - 2) = A + 2$
$A = B + 4$
Sub: $2B - 4 = (B + 4) + 2$
$2B - 4 = B + 6$
$B = 10$
$A = B + 4$
$= 10 + 4 = 14$
Total number of students is 24. (Excellence)

4. d = drink and c = packet of chips
$d = c + 2.5$ and $2d + 4c = 17$
sub: $2(c + 2.5) + 4c = 17$
$2c + 5 + 4c = 17$
$6c = 12$
$c = \$2$
$d = 2 + 2.5 = \$4.50$ (Excellence)

PRACTICE

5. x = adults and y = children
$x + y = 370$ and $8x + 5y = 2330$
$y = 370 - x$
sub: $8x + 5(370 - x) = 2330$
$8x - 1850 - 5x = 2330$
$3x = 480$
$x = 160$
160 adults paid for tickets. (Excellence)

6. let x = adults and y = children
$x + y = 8$ and $12x + 5y = 80$
$y = 8 - x$
sub: $12x + 5(8 - x) = 80$
$12x + 40 - 5x = 80$
$7x = 40$
$x = \frac{40}{7} = 5.71$
Round up from 5.33 to 6. A minimum of 6 adults is needed. (Excellence)

7. $x + y = 10$ and $7x + 8y = 200$
$x = 10 - y$
sub: $7(10 - \text{y}) + 8\text{y} = 200$
$70 - 7y + 8y = 200$
$y = 130$
$x = 10 - 130 = -120$ (Excellence)

8. c = cycling ticket, t = track ticket
$65c + 85t = 1095$ and $c + t = 15$
$c = 15 - t$
sub: $65(15 - \text{t}) + 85\text{t} = 1095$
$975 - 65t + 85t = 1095$
$20t = 120$
$t = 6$
$c = 15 - 6 = 9$ (Excellence)

9. s = seconds, x = export
$2s + 4x = 39.70$ and $3s + x = 39.80$
$x = 39.80 - 3s$
sub: $2\text{s} + 4(39.80 - 3\text{s}) = 39.70$
$-10s = -119.50$
$s = 11.95$
$x = 39.8 - 3 \times 11.95 = 3.95$ (Excellence)

10. $y = 40x + 175$ and $2y = 85x + 290$
sub: $2(40x - 175) = 85x + 290$
$80x + 350 = 85x + 290$
$60 = 5x$
$x = 12$
Year the same is $2005 + 12 = 2017$ (Excellence)

11. $20B + 35T = 24000$ and $B + T = 800$
$B = 800 - T$
sub: $20(800 - T) + 35T = 24000$
$16000 - 20T + 35T = 24000$
$15T = 8000$
$T = 533.3g$ (1 dp)
$B = 800 - 533.3 = 266.7g$ (1 dp) (Excellence)

12. $80H + 64C = 480000$ and $H = C + 1500$
sub: $80(\text{C} + 1500) + 64\text{C} = 480000$
$80\text{C} + 120000 + 64\text{C} = 480000$
$144\text{C} = 360000\text{C} = \$2500$
$\text{H} = 2500 + 1500 = \$4000$ (Excellence)

WORD QUESTIONS

STEPS:

1. Give each variable a letter, the letter will ALWAYS represent a number.
   E.g. R will represent the *number of* Rock CDs.
2. If a calculation is required, substitute the numbers given in the question into your equations.
   E.g. h is the height of the ball. The question tell you the ball is on the ground so substitute 0 in for h (height is zero).
3. You now have enough information to either solve the problem or explain your answer.

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OLD NCEA QUESTIONS

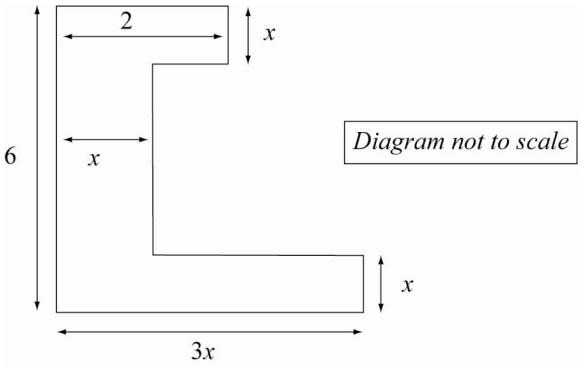
1. Ari spent $45 buying some CDs in a sale. He bought R Rock CDs and B Blues CDs.
Ari writes an equation for the amount he spent as: $2R + B = 45$.
Explain the terms of the equation

2. Tom and his son Tane are throwing a ball to each other on the deck of their house.
Tane misses the ball and it falls to the ground.
The path of the ball can be modelled by the equation $h = -t^2 + 2t + 8$ where t is the time in seconds since the ball is thrown and h is the height in meters above the ground at any time t.

a. How long after it is thrown will it hit take to hit the ground?
Explain what you are calculating at each step of your answer.

b. How much higher does the ball rise above the height of the point from which it is thrown?
Explain what you are calculating at each step of your answer.

3. Joey needs to make a path from the front of his house to the back as shown in the diagram below.
The width of the path is in metres.
Jim only has enough to make a path with a total area of 9 m^2.
Form equations and use these to find the width of the concrete path around his house.



PRACTICE QUESTIONS

4. A person in a hot air balloon dropped a hat accidently over the side of the basket. The height of the hat was modelled by the equation:

$h = 180 - 5t^2$

Where h = height of the hat above the ground (in m) and t = time since the hat was dropped (in seconds). Use the equation to find:

a. The height of the hot air balloon above the ground when the hat was dropped
Explain what you are calculating at each step of your answer.

b. The time, from when it was dropped, for the hat to hit the ground
Explain what you are calculating at each step of your answer.

5. 678 people paid a total of $9870 to get into a volleyball game. If n represents the number of adults at the game, then we can write the expression: $8n + 5(678 - n) = 9870$. Explain what the expression $(678 - n)$ represents.

6. Mr and Mrs Cameron had a possum problem on their farm. The possums came from other farms to eat their trees and so a deadly disease was introduced to get rid of the nasty creatures. Each day they counted the number of possums up their trees. The number of possums was modelled by the equation:

$P = -n^2 + 13n + 44$, where P is the number of possums and n = day number. Explain why n cannot be greater than 15.

7. Henry saved $5000 for a snowboarding trip to France. He wanted to board for as many days as possible. Each day pass cost $125. Travel, food and accommodation cost $3100. Write an equation and use it to tell how many days Henry is able to board with that saved money.

8. Katie and Sarah are sisters. Katie is 10 and Sarah is 4 years younger. Their favourite number is 896. Form a relevant equation and use it to find how many years it will take until Katie's and Sarah's ages, when multiplied together make 896.

9. At the rowing world champs 40 years ago, the average number of rowing teams per discipline was three times the total number of disciplines. At the Karapiro world champs in 2010, there were 10 more disciplines than 40 years ago and three times as many teams/competitors per discipline. There were a total of 3375 teams/competitors at the 2010 world champs. Write at least one equation to model this situation. Use the model to find the number of disciplines that were contested 40 years ago.

Study Tip:

Challenge Yourself

***Don't**: practice questions that are **too easy** – your brain discounts the information as not important.*
***Do**: **Challenge yourself** – the information is **more likely to stick** (and you learn how to solve difficult problems!)*

ANSWERS

NCEA

1. $2R$ is the amount spent on rock CDs or \$2.00 per Rock CD
$1D$ is the amount spent on blues CDs or \$1.00 per Blues CD
\$45 is the total amount spent.
(Achieved - one correct)
(Merit - two correct)
(Excellence - three correct)

2.

a.
$$(10+x)(6+x)=896$$
$$x^2+10x+6x+60=896$$
$$x^2+16x-836=0$$
$$(x+38)(x-22)=0$$
$$x=-38 \text{ or } x=22$$

The ball will hit the ground when h = 0 so solve:
$$-t^2+2t+8=0$$
$$t^2-2t-8=0$$
$$(t+2)(t-4)=0$$
$$t=-2 \text{ or } 4$$

As t cannot be negative, the ball must hit the ground after 4 seconds. (Merit)

b. The highest point is at $t=\dfrac{-2+4}{2}=1$. This is because 1 is half way between the -2 and 4 second solutions found in part (a).
When $t=1,\ h=-(1)^2+2(1)+8=9$
so the ball rises 1 m higher than when it is thrown, from 8 m to 9 m.
(Excellence)

3.
$$\text{Area}=2\times x+3x\times x+(6-2x)\times x$$
$$=2x+3x^2+6x-2x^2$$
$$=x^2+8x$$
$$x^2+8x=9 \text{ m}^2$$
$$x^2+8x-9=0$$
$$(x-1)(x+9)=0$$
$$x-1=0 \text{ and } x+9=0$$
$$x=1 \text{ and } \cancel{x=-9}$$
x = -9 is not a valid solution so Jim's path must be 1 m wide. (Excellence)

PRACTICE

4.

a. The hat was dropped at t = 0 and so the height of the balloon will be the same as the height of the hat at t = 0.
$$h=180-5(0)^2=180 \text{ m}$$ (Merit)

b. The hat hit the ground when h = 0 so solving for that will give the time taken to fall.
$$180-5t^2=0$$
$$36-t^2=0$$
$$t^2=36$$
$$t=6 \text{ seconds}$$
(Merit)

5. This looks like a substitution into a simultaneous equation. If n is the number of adults then it can be assumed that $(678-n)$ is the number of other people who are attending (children/students). 678 would represent the total number of people at the event. (Merit)

6. The equation; $P=-n^2+13n+44$ shows that when n^2 is greater than $13n+44$, P will be negative, as n^2 is negative. This means that there will be a negative number of possums after 15 days and this is impossible. (Merit)

7.
$$5000-3100-125x\geq 0$$
$$1900-125x\geq 0$$
$$1900\geq 125x$$
$$15.2\geq x$$
The most days he can board is 15.
(Excellence)

8. Let x be the number of years to wait.
$$(10+x)(6+x)=896$$
$$x^2+10x+6x+60=896$$
$$x^2+16x-836=0$$
$$(x+38)(x-22)=0$$
$$x=-38 \text{ or } x=22$$
Time cannot be negative so 22 years is required before their ages multiply together to make 896.
(Excellence)

9. c=no. of teams/competitors 40 years ago
x=no. of disciplines 40 years ago
s=no. of disciplines in 2010
$$\frac{c}{x}=3x$$
$$x+10=s$$
$$\frac{3375}{s}=3(3x)=9x$$
$$\frac{3375}{x+10}=9x$$
$$3375=9x(x+10)$$
$$3375=9x^2+90x$$
$$9x^2+90x-3375=0$$
$$x^2+10x-375=0$$
$$(x-15)(x+25)=0$$
$$x=15 \text{ or } x=-25$$
40 years ago there were 15 disciplines at the rowing world champs (-25 is not valid in this situation)
(Excellence)